F. Méot, BNL, Sept. 2009
C-AD/AP/452

# Spin tracking simulations in AGS based on ray-tracing methods
## - bare lattice, no snakes -

A work performed at BNL in September and October 2009, in collaboration with
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## Abstract

This Note reports on the first simulations of and spin dynamics in the AGS using the ray-tracing code Zgoubi.

It includes lattice analysis, comparisons with MAD, DA tracking, numerical calculation of depolarizing resonance strengths and comparisons with analytical models, etc. It also includes details on the setting-up of Zgoubi input data files and on the various numerical methods of concern in and available from Zgoubi.

This work has been followed by further spin dynamics studies in presence of the AGS helical snakes, see Note CAD/AP/453.

# Contents

# 1 Introduction

Simulations of crossing and neighboring of spin resonances in AGS ring, bare lattice, without snake, have been performed, in order to assess the capabilities of Zgoubi in that matter, and are reported here. This yields a rather long document. The two main reasons for that are, on the one hand the desire of an extended investigation of the energy span, and on the other hand a thorough comparison of Zgoubi results with analytical models as the "thin lens" approximation, the weak resonance approximation, and the static case.

Section 2 details the working hypothesis : AGS lattice data, formulae used for deriving various resonance related quantities from the ray-tracing based "numerical experiments", etc.
Section 3 gives inventories of the intrinsic and imperfection resonances together with, in a number of cases, the strengths derived from the ray-tracing.
Section 4 gives the details of the numerical simulations of resonance crossing, including behavior of various quantities (closed orbit, synchrotron motion, etc.) aimed at controlling that the conditions of particle and spin motions are correct.
In a similar manner Section 5 gives the details of the numerical simulations of spin motion in the static case : fixed energy in the neighboring of the resonance.
In Section 6, weak resonances are explored, Zgoubi results are compared with the Fresnel integrals model.
Section 7 shows the computation of the $\vec{n}$ vector in the AGS lattice and tuning considered.
Many details on the numerical conditions as data files etc. are given in the Appendix Section, pages A and sqs.

# 2 Preliminary data, working hypothesis

This section details the working hypothesis, from both viewpoints of lattice data and of ray-tracing basic dynamics outputs.

## 2.1 Lattice

Tab. 1 displays the general optical parameters as obtained from MAD8, various input/output files of concern are reproduced in App. A.

Zgoubi optics file is translated from MAD8 "survey" output (a translator is available), parameter values so obtained are given in the middle col. ("Brute from MAD8) in Tab. 1 for comparison with MAD ones. A typical "zgoubi.dat" input file is displayed in App. B.1.

The $\approx 3.2$ cm difference in orbit length arises from the treatment of the reference closed orbit which does not coincide with the "Optimum Closed Orbit" (so-called "OCO"), in the present work. The theoretical length of the OCO is 807.09 m, this has been settled in further works, see PAC and IPAC Conference papers, 2010, 2011. The $\approx 9\%$ difference in $D_x$ remains to be explained.

Table 1: AGS parameters. The middle column "Brute from MAD8" gives the working conditions in the numerical experiments reported in the following sections. The right most column shows how some parameter have to evolve so to obtain zero closed orbit at main bend ends.

| | | MAD8 | Ray-tracing[1] (Hard edge) | |
|---|---|---|---|---|
| | | | Brute from MAD8 | Adjusted c.o. length[4] |
| Reference momentum | (relative) | 1 | 1 | 1.00280 |
| Orbit length | (m) | 807.07564 | 807.04378 | 807.07564 |
| Perimeter of polygon[5] | (m) | | 807.06007 | |
| Qx, Qy | | 8.71060, 8.76438 | [8].71195, [8].76346[2] | [8].65346, [8].76816 |
| Q'x, Q'y | | -22.7340, 1.7343 | -20.9864, 1.7943 | |
| $\alpha$, $\sqrt{1/\alpha}$ | | 0.01401, 8.44965 | 0.01400, 8.45102 | |
| *Periodic functions at "Begin AGS" :* | | | | |
| $\beta_x,\ \beta_y$ | (m) | 19.785, 11.701 | 19.8432, 11.6751 | |
| $\alpha_x,\ \alpha_y$ | | -1.585, 1.037 | -1.588, 1.033 | |
| $D_x, D'_x$ | (m,-) | 2.211, 0.154 | 2.034, 0.144 | |
| closed orbit, $x_{co}, x'_{co}$ | mm, mrad | 0, 0 | -6.43 -0.50[1,3] | $\approx 0$, $\approx 0$ |

[1] Combined function dipoles are simulated using straight axis multipoles and $d^nB/dx^n$ gradients, the "MULTIPOL" keyword, whereas MAD uses SBEND.
[2] Obtained from either coordinate interpolation or multiturn Fourier analysis.
[3] The c.o. does not coincide with the "OCO", here.
[4] Closed orbit adjusted using FIT procedure, cf. App. B.3. Tunes changed by $Q' \times dp/p$
[5] The polygon is comprised of the axis of optical elements, all straight.

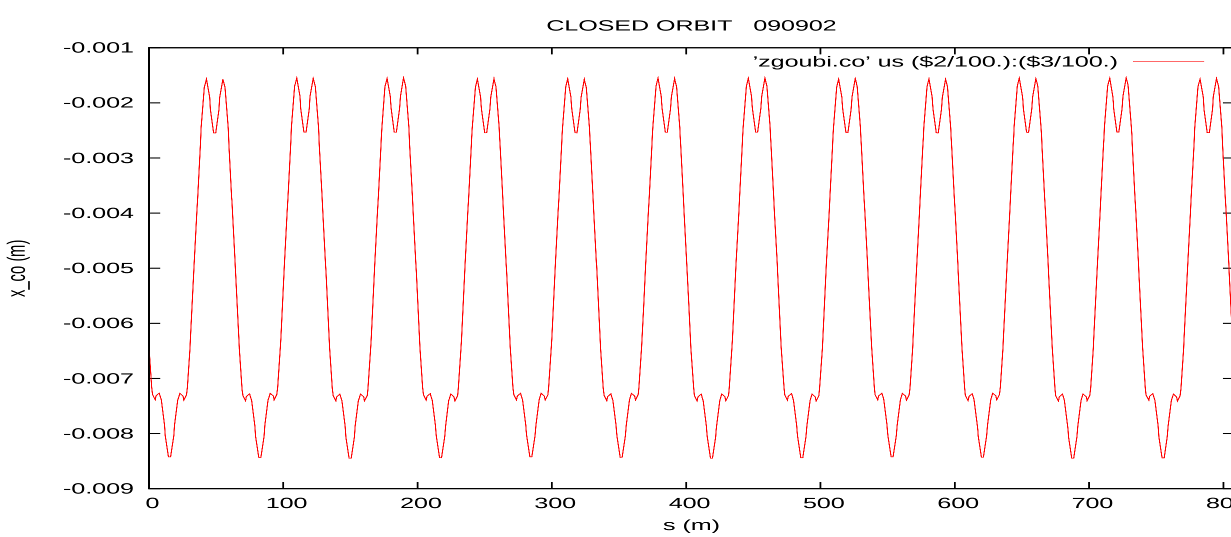



Figure 1: Closed orbit, induced by straight axis in bends (all hard-edge model).

**A superimposition of MAD/TWISS and Zgoubi outputs :**

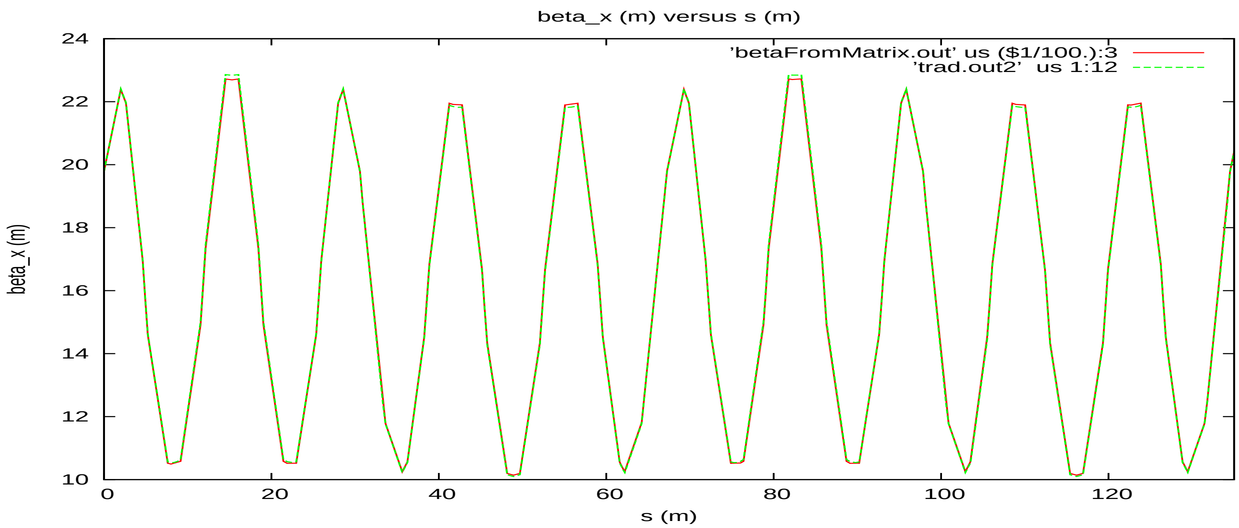



Figure 2: $\beta_x$ over the first two superperiods.

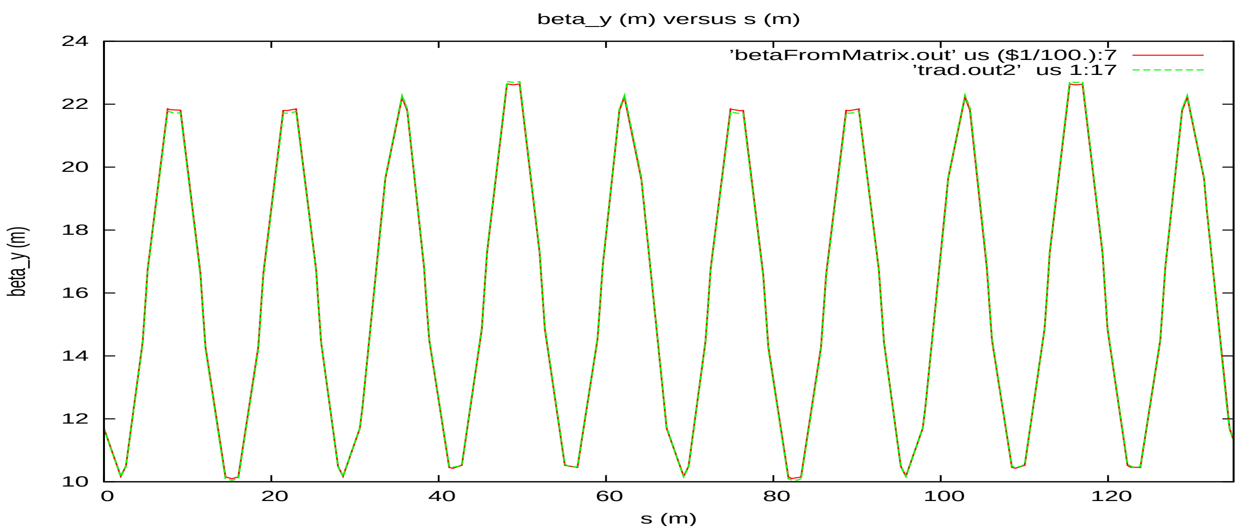



Figure 3: $\beta_y$ over the first two superperiods.

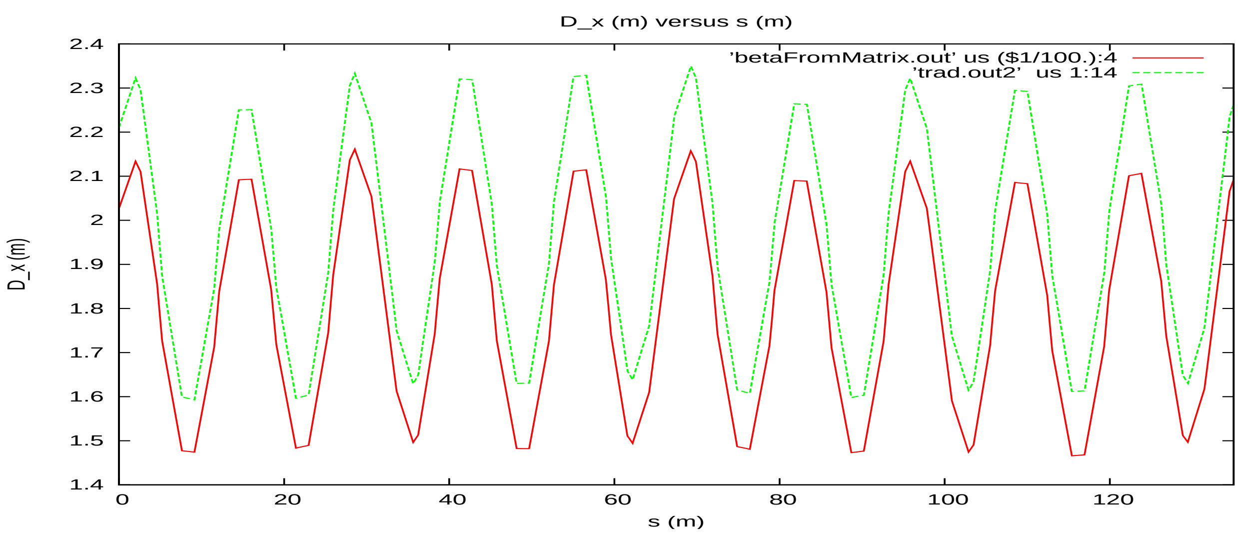



Figure 4: $D_x$ over the first two superperiods.

## 2.2 DA. Long term tracking

Considering the importance of a good model of the ring dynamics if spin is to be tracked, we push further the investigation on the lattice we are working with, and of the ray-tracing outputs, by checking large excursion behavior, in terms of maximum stable amplitudes and DA. Long term tracking behavior is also tested.

### 2.2.1 Maximum stable amplitudes

Figs. 5, 6 show sample results of maximum stable amplitude tracking, 1000 turns.

It can be observed that the numerical integration in the horizontal case (Fig. 5) does not exhibit noticeable spiraling, an indication of correct symplectic behavior.

In the case of the vertical stability limit tracking (Fig. 6), coupling induces large horizontal motion, at the origin of spreading of the vertical invariant into a donut phase portrait ; the origin (e.g., width of non-linear coupling resonance, lack of tracking precision...) remains to be determined.

### 2.2.2 Dynamic aperture

Fig. 8 gives the dynamic aperture of the ring for $\delta p/p = 0, \ \pm 1, \pm 2, \pm 3\%$. These $(x, z)$ limits are obtained by scanning the x-axis, step size 3 cm, and looking, at each $x$-step, for the maximum vertical stable amplitude upon 1000-turn tracking (namely, doing what is illustrated in Fig. 6), with 2 mm precision on that $z$-limit. The operation is performed repeatedly for the various momenta.

### 2.2.3 Symplecticity tests

Long term tracking may be required in assessing depolarizing effects in AGS or RHIC. Fig. 9 displays some sample results, showing very good behavior. The integration step size is $\sim 1$ cm in all optical elements.

In the case of RHIC, it *the step size can be a little larger* given that the dipoles are not combined function. It has been checked that 1.5 cm at top energy (where spin moves the fastest in the dipoles) does ensure in addition convergence of the solution of $\vec{S}' = \vec{x}\vec{\omega}$.

Spin has been tracked in AGS step by step as well during this $n \approx 5\,10^5$-turn motion tracking. Zgoubi calculates independently the three components, $S_x, S_y, S_z$. It comes out that $|\vec{S}|^2 = S_x^2 + S_y^2 + S_z^2 \equiv 1$, from beginning to end. Similar behavior in RHIC at top energy with 1.5 cm step size.

**Horizontal stability limit**

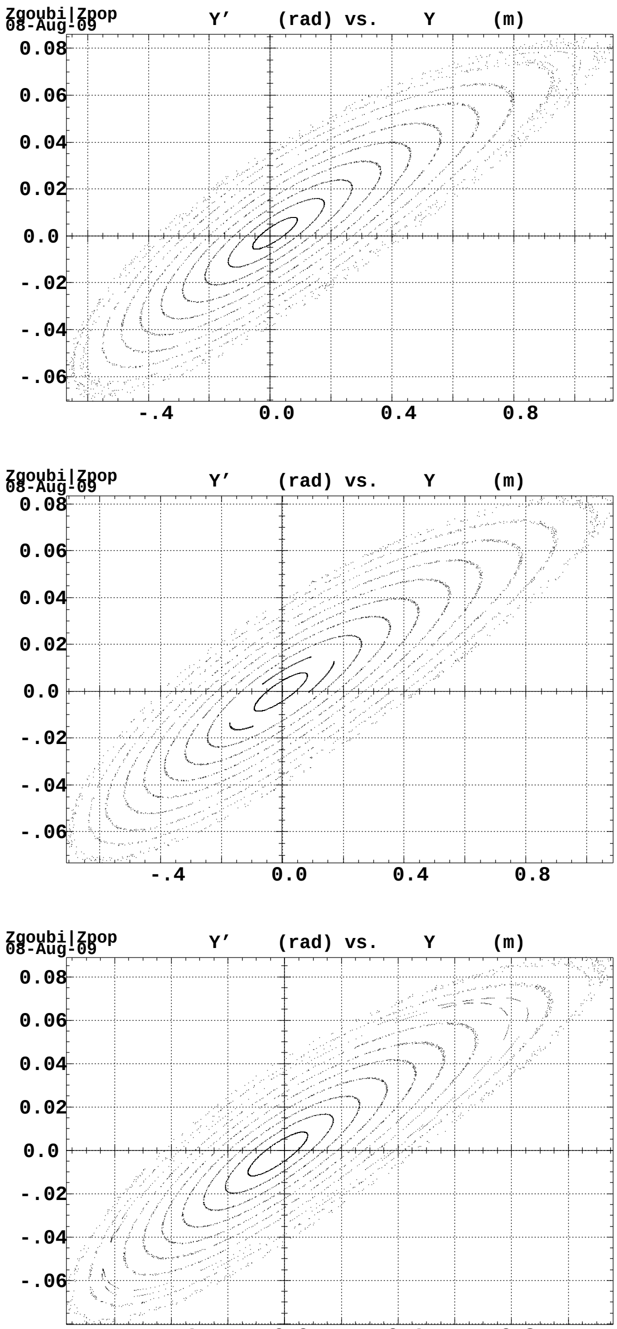



Figure 5: Maximum horizontal stable amplitude, 1000 turns in the ring, case of zero vertical emittance.
From top to bottom : $\delta p/p = +0.01,\ 0,\ -0.01$.
Hard edge optical elements.

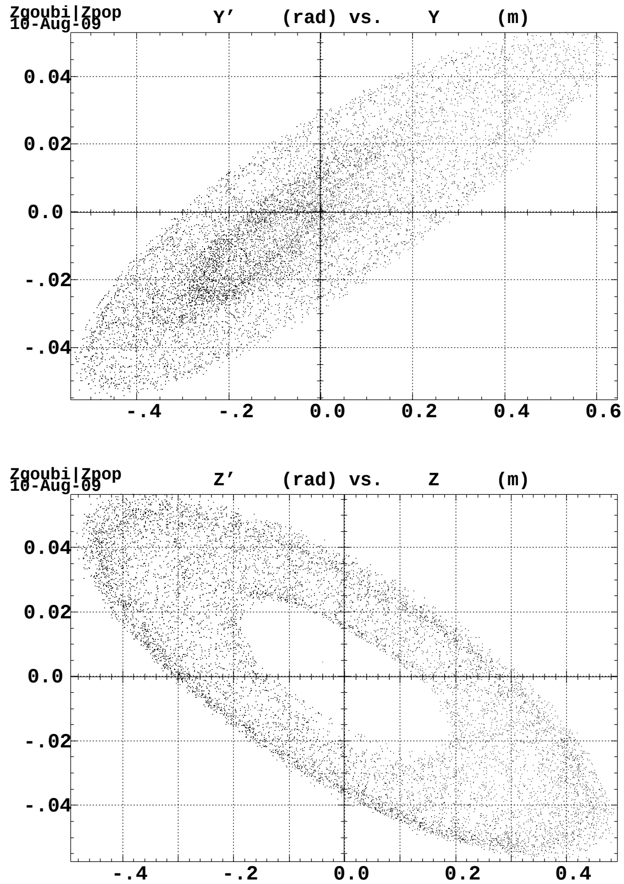



Figure 6: Maximum vertical stable amplitude, 1000 turns, showing H-V emittance exchange, $\delta p/p = 0$.
Top plot : coupling induced $x$-motion ; bottom plot : vertical donut at maximum stable amplitude.
Hard edge optical elements.

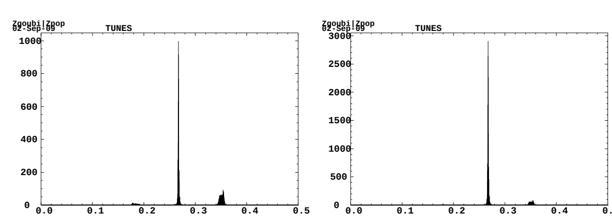



Figure 7: x-x' and z-z' spectra (left, right), exhibiting coupling.

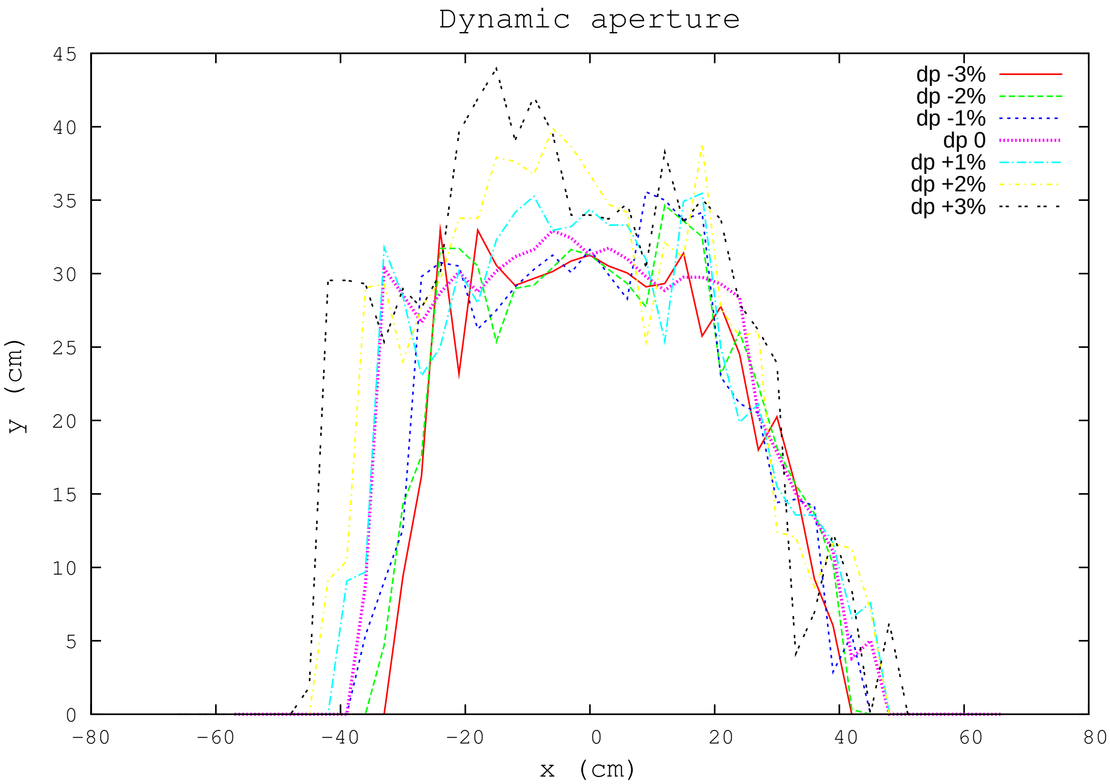
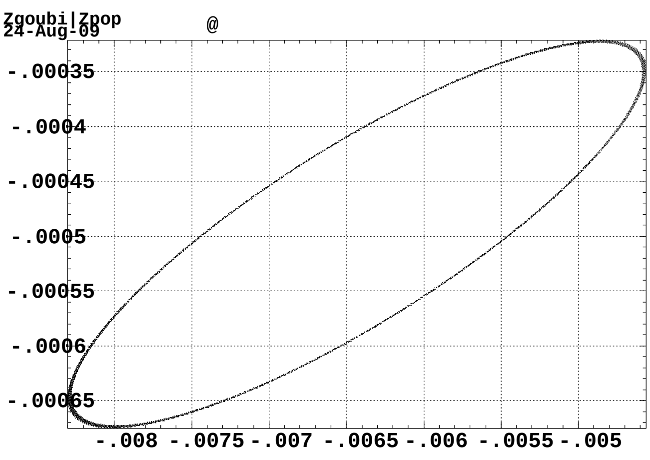



Figure 8: Dynamical apertures at $\delta p/p = 0, \ \pm 1, \ \pm 2, \ \pm 3\%$. 3 cm step in $x$, 2 mm precision on 1000-turn maximum vertical stable amplitude.
Hard edge optical elements.



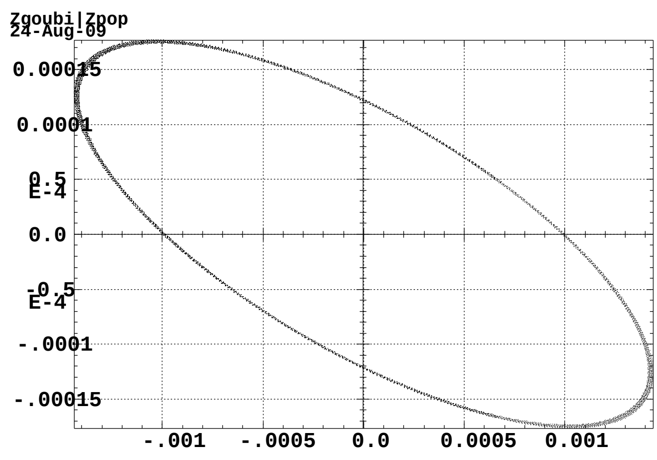




Figure 9: $4\,10^5$-turn tracking for a particle launched on $\epsilon_x/\pi = \epsilon_z/\pi = 0.35$ mm.mrad, at fixed rigidity, observed at "Begin AGS".
There is no substantial broadening of the invariants, nor visible spiraling.
Note : a matching of optical functions yields, $\beta_x = 19.8$ m, $\alpha_x = -1.58$, $\beta_z = 11.7$ m, $\alpha_z = 1.03$, consistent with Tab. 1 data.
Hard edge optical elements.

## 2.3 Quantities and formulas used in these spin tracking studies

Classical formulae which will be used in the "numerical experiments" to follow are recalled below, together with various numerical data values[1].

### 2.3.1 Working hypotheses

The crossing speed writes

$$\alpha = G\frac{d\gamma}{d\theta} = G\,\frac{1}{2\pi}\frac{\Delta E}{M_0} \tag{1}$$

with $M_0 = 938.27203$ MeV, $G = 1.7928474$. In the numerical simulations we take
$\hat{V} = 290$ kV
$\phi_s = 30$ degrees or $150$ degrees
hence

$$\Delta E = 145 \text{ keV/turn} \quad \text{and} \quad \alpha = 4.4096356\,10^{-5}$$

Besides,
$\dot{B} = \Delta E\,/\,(2\pi R\rho) = \Delta E\,/\,(\mathcal{C}\rho)$
The ring circumference is (Tab. 1)
$\mathcal{C} = 807.04378$ m
whereas $1/\rho = B/B\rho = 0.01171255$ (see dipole component in 'SBEN' type of magnets, zgoubi.dat file in App. B.1). (Note : $\rho = 85.378504$, packing factor $R/\rho = 9.4525406$). This yields

$$\dot{B} = 2.104 \text{ T/s}$$

### 2.3.2 Asymptotic depolarization

Crossing of an isolated resonance entails asymptotic depolarization given by the Froissard-Stora formula,

$$\frac{p_{final}}{p_{initial}} = 2\exp(-A^2)\,-\,1 \qquad = 2\exp(-\frac{\pi}{2}\frac{|J_n|^2}{\alpha})\,-\,1 \tag{2}$$

### 2.3.3 Depolarization, static

Free oscillation of polarization vector $\vec{S}$ around arbitrary local precession vector $\vec{\omega}$, at fixed energy, starting with $\vec{S} \equiv \vec{S}_z$, satisfies

$$\frac{(-)}{\sqrt{1-|C|^2}}\frac{dC}{d\theta} = J_n e^{j\Delta\theta} \text{ with C of the form } e^{j\mu\theta} \text{ and } \rho = constant \text{ yielding } \rho^2 = \frac{1}{1+\Delta^2/|J_n|^2} \tag{3}$$

with

$$\Delta = \text{ distance to the resonance } = \gamma G - (n\times M - \nu_z)$$

$$\bar{S}_z^2 = 1 - |\rho|^2 \quad \text{yields} \quad \bar{S}_z^2 = \frac{1}{1+|J_n|^2/\Delta^2} \tag{4}$$

with $\bar{S}_z$ the average value of $S_z$. This allows computing $|J_n|$ from the numerical value of $S_z$, following

$$|J_n|^2 = \Delta^2/\left(\frac{1}{\rho^2}-1\right) \tag{5}$$

In particular,
1% depolarization ($\bar{S}_z = 0.99$) corresponds to $\Delta = \gamma G - n = 7|J_n|$ (an energy band $\pm\delta\gamma = \pm 7|J_n|/G$)
86.6% depolarization ($\rho = 0.5$) corresponds to $\Delta = \gamma G - n = \sqrt{3}|J_n|$

[1] Ref. Gérard Leleux, Traversée des résonances de dépolarisation, unpublished, SATURNE, Saclay, 15 Février 1992.

#### 2.3.4 Weak resonance, Fresnel integral approximation

Reference : Gérard Leleux, Traversée des résonances de dépolarisation, unpublished, SATURNE, Saclay, 15 Février 1992.

Depolarization in the case of weak resonances (i.e., $p_f/p_i \approx 1$) is given by

• upstream of the resonance $(\theta < 0)$ : $(p(\theta)/p_i)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2 \left[(0.5 - C(-\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 - S(-\theta\sqrt{\frac{a}{\pi}}))^2\right]$ (6)

• downstream of the resonance $(\theta > 0)$ : $(p(\theta)/p_i)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2 \left[(0.5 + C(\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 + S(\theta\sqrt{\frac{a}{\pi}}))^2\right]$

where

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)\,dt, \qquad S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)\,dt$$

are the Fresnel integrals.

Note that

$$p(\theta)/p_i \xrightarrow{\theta\to\infty} 1 - \frac{\pi}{\alpha}|J_n|^2 \tag{7}$$

i.e., Froissard-Stora formula (Eq. 2) taken to first order in $|J_n|^2/\alpha$. The approximation holds in the limit that higher order terms can be neglected, i.e. $\pi|J_n|^2/\alpha$ is small enough compared to 1.

In Section 6, Zgoubi is tested against the weak-resonance approximation.

#### 2.3.5 Resonance strength, theoretical

It can be computed from MAD type of output,using the "thin lens model" below. This allows comparison with ray-tracing results. The following formulae hold.

Imperfection, thin lens approximation :

$$J_n = \frac{1+\gamma G}{2\pi}\Sigma_{Qpoles} \left\{ \begin{array}{c} \cos(\gamma G\alpha_i) \\ \sin(\gamma G\alpha_i) \end{array} \right\} (KL)_i z_{co,i} \tag{8}$$

with $\alpha_i$ cumulated bend angle, $z_{co,i}$ the closed orbit amplitude, at quadrupole $i$ with strength $(KL)_i$.

Intrinsic, thin lens approximation :

$$J_n^{\pm} = \frac{1+\gamma G}{4\pi}\Sigma_{Qpoles} \left\{ \begin{array}{c} \cos(\gamma G\alpha_i \pm \psi_i) \\ \sin(\gamma G\alpha_i \pm \psi_i) \end{array} \right\} (KL)_i \sqrt{\beta_{z,i}\epsilon_z/\pi} \tag{9}$$

with

$$\psi_i = \int_0^{\theta_i} \frac{ds}{\beta_z}, \qquad '+' \text{ sign for } \gamma G + \nu_z - n = 0, \qquad '-' \text{ sign for } \gamma G - \nu_z - n = 0$$

and $(KL)_i$ strength of quadrupole $i$ located at $\theta_i$.

# 3 Inventory of spin resonances, including spin tracking results

Two inventories of the intrinsic resonances are given in Tables 2, 3 respectively, below. They include the numerical value of the resonance strengths as obtained for the numerical ray-tracing, in both dynamic (crossing) and static cases. The difference between both (a factor 3-4) remains to be elucidated.

## 3.1 Intrinsic resonances

Table 2: Intrinsic resonances. $\nu_z = 8.76346$, $M = 12$ superperiods. $A^2 = \frac{\pi}{2}\frac{|J_n|^2}{\alpha}$.

*Intrinsic resonances, systematic, $kM \pm \nu_z$, $M = 12$, classed by energy.*

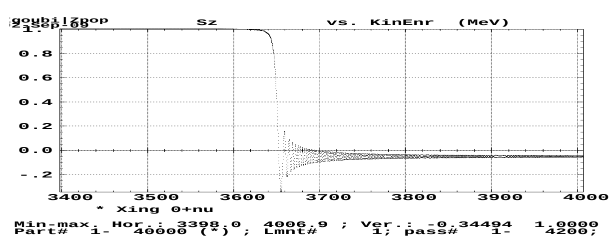
| $kM \pm \nu_z$ | $\gamma G$ | kin. E (GeV) | $B\rho$ (T.m) | $\epsilon_z/\pi$ ($10^{-6}$) | $A^2$ | $\lvert J_n\rvert^2$ ($10^{-6}$) | $\frac{A^2}{\epsilon_z/\pi}$ ($10^6$) | $\frac{\lvert J_n\rvert^2}{\epsilon_z/\pi}$ Thin lens model | $\frac{\lvert J_n\rvert^2}{\epsilon_z/\pi}$ ZGOUBI dyn. | $\frac{\lvert J_n\rvert^2}{\epsilon_z/\pi}$ ZGOUBI stat.[a] | Final $P_z$ (dyn.) | $\lvert 4\nu_z^2 - (kM)^2\rvert$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 +qz | 8.76345 | 3.64801 | 14.9746 | 0.0017 | 0.0071 | 0.200 | 3.5576 | 125 | 100.0 | 390 | 0.9858 | 307 |
| | | | | 0.01 | 0.0357 | 1.003 | 3.5731 | | 100.3 | | 0.9298 | |
| | | | | 0.017 | | | | | | 455 | | |
| | | | | 0.05 | 0.1788 | 5.020 | 3.5765 | | 100.4 | | 0.6725 | |
| | | | | 0.2 | 0.7467 | 2.096 | 3.7338 | | 104.8 | | -0.0522 | |
| 24 -qz | 15.2365 | 7.03564 | 26.4134 | 0.1 | 0.0011 | 0.0295 | 0.1051 | 0.30 | 0.30 | | 0.9979 | 269 |
| 12 +qz | 20.7634 | 9.92811 | 36.1110 | 0.01 | 0.0026 | 0.072 | 1.2826 | 39 | 37 | | | 163 |
| 12 +qz | 20.7634 | 9.92811 | 36.1110 | 0.1 | 0.1290 | 3.621 | 1.2897 | | 37 | | 0.7590 | |
| 12 +qz | 20.7634 | 9.92811 | 36.1110 | 2 | 2.5431 | 71.39 | 1.2716 | | 37 | | -0.8428 | |
| 36 -qz | 27.2365 | 13.3157 | 47.4432 | 0.05 | 0.5494 | 15.42 | 10.989 | 320 | 309 | | 0.1545 | 989 |
| 24 +qz | 32.7634 | 16.2082 | 57.1088 | 2 | 0.0737 | 2.070 | 0.0368 | 0.90 | 1.04 | | 0.857 | 269 |
| | | | | 30 | 1.0113 | 28.39 | 0.0337 | | 0.95 | | -0.269 | |
| 48 -qz | 39.2365 | 19.5958 | 68.4229 | 0.125 | 0.0463 | 1.299 | 0.3704 | 4.90 | *10.4* | 27 | 0.9095 | 1997 |
| | | | | 0.25 | 0.0986 | 2.768 | 0.3945 | | 11.1 | | 0.812 | |
| | | | | 0.5 | 0.1913 | 5.370 | 0.3826 | | 10.7 | | 0.6515 | |
| | | | | 1 | 0.3895 | 10.94 | 0.3895 | | 10.9 | | 0.3545 | |
| | | | | 2 | 0.8073 | 22.67 | 0.4036 | | 11.3 | 30 | -0.1078 | |
| 36 +qz | 44.7634 | 22.4883 | 78.0800 | 0.0001 | 0.0066 | 0.186 | 66.118 | 1950 | 1856 | | 0.9868 | 989 |
| | | | | 0.002 | 0.1329 | 3.733 | 66.480 | | 1937 | 4997 | 0.751 | |
| | | | | 0.02 | 1.4318 | 40.19 | 71.592 | | 2010 | 5560 | -0.522 | |
| 60 -qz | 51.2365 | 25.8759 | 89.3879 | | | | | 73400 | | | | 3293 |

(a) Emittances in static case may slightly differ from that given in the '$\epsilon_z/\pi$' column, exact values can be found in Sec. 5.

**Thin lens model,** data obtained using MAD output and :

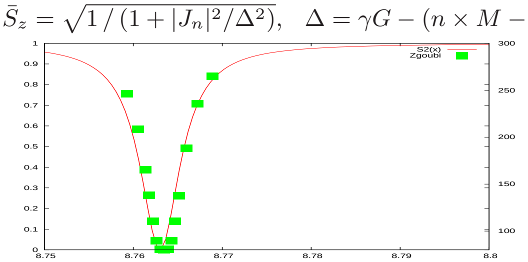
$$J_n^{\pm} = \frac{1+\gamma G}{4\pi}\Sigma_{Qpoles}\left\{\begin{array}{c}\cos(\gamma G\alpha_i \pm \psi_i)\\ \sin(\gamma G\alpha_i \pm \psi_i)\end{array}\right\}(KL)_i\sqrt{\beta_{z,i}\epsilon_z/\pi}$$

**ZGOUBI** , data obtained using :




or

$$\bar{S}_z = \sqrt{1/\left(1+|J_n|^2/\Delta^2\right)}, \quad \Delta = \gamma G - (n \times M - \nu_z)$$



### 3.2 Imperfection resonances

A closed orbit is excited using a vertical kicker (DVCA02) ; the fourth column in Tab. 3 gives the maximum amplitude of the resulting closed orbit around AGS.

Table 3: Closed orbit resonances.

| $\gamma G$ | kin. E (GeV) | $B\rho$ (T.m) | $\hat{z}$ (mm) | $A^2$ | $\lvert J_n\rvert^2$ ($10^{-5}$) | $A^2/\hat{z}^2$ | $\lvert J_n\rvert^2/\hat{z}^2$ Thin lens model | $\lvert J_n\rvert^2/\hat{z}^2$ Zgoubi dyn. | Final $P_z$ |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 1.678437 | 8.147990 | | | | | | | |
| 6 | 2.201779 | 9.995557 | | | | | | | |
| 7 | 2.725121 | 11.81217 | | | | | | | |
| 8 | 3.248463 | 13.61023 | | | | | | | |
| 9 | 3.771804 | 15.39624 | .139 | | | | 4.75 | | |
| | | | 2.77 | 1.224737 | 3.4381555 | 159618 | | 4.48 | -0.412 |
| 10 | 4.295146 | 17.17396 | | | | | | | |
| 11 | 4.818487 | 18.94571 | | | | | | | |
| 12 | 5.341830 | 20.71305 | 13.84 | 0.6379532 | 1.7909011 | 3330 | .031 | 0.094 | 0.05674 |
| 13 | 5.865171 | 22.47699 | 13.84 | 0.5469143 | 1.5353313 | 2855 | .019 | 0.080 | 0.15745 |
| 14 | 6.388514 | 24.23830 | | | | | | | |
| 15 | 6.911855 | 25.99749 | | | | | | | |
| 16 | 7.435197 | 27.75499 | | | | | | | |
| 17 | 7.958539 | 29.51107 | | | | | | | |
| 18 | 8.481881 | 31.26600 | | | | | | | |
| 19 | 9.005222 | 33.01994 | | | | | | | |
| 20 | 9.528564 | 34.77305 | | | | | | | |
| 21 | 10.05191 | 36.52545 | | | | | | | |
| 22 | 10.57525 | 38.27723 | | | | | | | |
| 23 | 11.09859 | 40.02848 | 13.84 | 1.734326 | 4.868705 | 905437 | .082 | 0.25 | -0.6468 |
| 24 | 11.62193 | 41.77927 | | | | | | | |
| 25 | 12.14527 | 43.52964 | | | | | | | |
| 26 | 12.66861 | 45.27966 | | | | | | | |
| 27 | 13.19196 | 47.02935 | 1.384 | 0.8763353 | 2.460102 | 457507 | 11.3 | 12.8 | -0.167355 |
| 28 | 13.71530 | 48.77875 | | | | | | | |
| 29 | 14.23864 | 50.52790 | | | | | | | |
| 30 | 14.76198 | 52.27681 | | | | | | | |
| 31 | 15.28532 | 54.02551 | | | | | | | |
| 32 | 15.80867 | 55.77403 | | | | | | | |
| 33 | 16.33201 | 57.52238 | | | | | | | |
| 34 | 16.85535 | 59.27057 | | | | | | | |
| 35 | 17.37869 | 61.01860 | | | | | | | |
| 36 | 17.90203 | 62.76652 | | | | | | | |
| 37 | 18.42537 | 64.51430 | | | | | | | |
| 38 | 18.94872 | 66.26198 | | | | | | | |
| 39 | 19.47206 | 68.00956 | | | | | | | |
| 40 | 19.99540 | 69.75704 | | | | | | | |
| 41 | 20.51874 | 71.50443 | | | | | | | |
| 42 | 21.04208 | 73.25175 | | | | | | | |
| 43 | 21.56543 | 74.99899 | | | | | | | |
| 44 | 22.08877 | 76.74614 | | | | | 9.0 | | |
| 45 | 22.61211 | 78.49324 | 0.028 | | | | 68.3 | | 0.99596 |
| | | | 0.275 | 0.2047056 | 0.57466214 | 2706852 | | 76.0 | 0.62962 |
| 46 | 23.13545 | 80.24029 | | | | | 4.3 | | |

**Thin lens model**, data obtained using MAD output and :

$$J_n = \frac{1+\gamma G}{2\pi}\Sigma_{Qpoles}\left\{\begin{array}{l}\cos(\gamma G\alpha_i)\\ \sin(\gamma G\alpha_i)\end{array}\right\}(KL)_i z_{co,i} \tag{10}$$

## I built a "Xing factory" from all that material, applicable to AGS and RHIC :

• Starting from a MAD/TWISS file,
a sheet is produced, which contains strengths, vertical closed orbit etc. :

```
 n   +/-Qz    Jn^2/zma^2 Nn^2/eps_z/pi,  zmax       vkick      2pi-2pi
  0 +8.7648   0.0      127.3933      0.0000E+00  0.0000E+00  0.2032E-10
 12 +8.7648   0.0       39.2391      0.0000E+00  0.0000E+00  0.2032E-10
 24 -8.7648   0.0        0.3019      0.0000E+00  0.0000E+00  0.2032E-10
 24 +8.7648   0.0        0.9011      0.0000E+00  0.0000E+00  0.2032E-10
 36 -8.7648   0.0      321.0361      0.0000E+00  0.0000E+00  0.2032E-10
 36 +8.7648   0.0     1937.6307      0.0000E+00  0.0000E+00  0.2032E-10
 48 -8.7648   0.0        4.87452     0.0000E+00  0.0000E+00  0.2032E-10
```

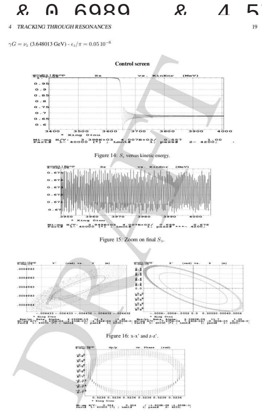
Strengths from lattice functions are computed using the regular thin-lens modelling :
- closed orbit : $(J_n/\hat z) = \frac{1+\gamma G}{2\pi}\Sigma_{Qpoles}\left\{\begin{array}{c}\cos(\gamma G\alpha_i)\\ \sin(\gamma G\alpha_i)\end{array}\right\}(KL)_i z_{co,i}\,/\,\hat z$
- intrinsic : $(J_n^\pm/\sqrt{\epsilon_z/\pi}) = \frac{1+\gamma G}{4\pi}\Sigma_{Qpoles}\left\{\begin{array}{c}\cos(\gamma G\alpha_i\pm\psi_i)\\ \sin(\gamma G\alpha_i\pm\psi_i)\end{array}\right\}(KL)_i\sqrt{\beta_{z,i}}$

Note that the long combined function AGS bends need be split into 4 pieces at least so to get the strengths in that table converged.

• Taking $n\pm\nu_z$ from that sheet, and including in addition working conditions for the tracking,
- under the form of a template zgoubi.dat file obtained from MAD translation,
- with specific values for $\hat V$, $\phi_s$, etc.
then Zgoubi will perform a scan of all resonances, one after the other.
Initial particle conditions are taken to be :
- closed orbit : $\hat z_{co} = \sqrt{8\,10^{-6}}/(J_n/\hat z_{co})$
- intrinsic : $\epsilon_y/\pi = \sqrt{8\,10^{-6}}/(J_n/\epsilon_y)$
with $\sqrt{8\,10^{-6}}$ in there just because $2\exp(-\frac{\pi}{2}\frac{|J_n|^2}{\alpha})-1\approx 0.5$ (given $\alpha\approx 4.41\,10^{-5}$) is a convenient value for appropriate accuracy when drawing $p_f/p_i$ from the tracking data.
The distance $\Delta$ upstream of the resonance where to start from is given by (in units of $J_n$)
$\Delta/|J_n| = S_z/\sqrt{1-S_z^2}$
taking for instance $S_z = 1-10^{-4}$, which translates into a number of turns using $\gamma G = \Delta + (n\pm\nu_z)$ and the acceleration rate, $dE$ per turn.

This tool produces the typical following table :

```
      &  Energy &   Qz    &   e_z/pi  &  p_init  & p_final  & |J_n|^2 &    |J_n|^2/ez  } \\
      &   (GeV) &         &  (1e-6)   &          &          &  (1e-6) & ZGOUBI  &   MAD \\
 0+Qz &  4.2359 & 0.76345 & 3.604E-02 &  1.000   & 0.7434   &  3.855  &  107.0  &  127.4
12+Qz &  10.516 & 0.76345 & 0.123     &  1.000   & 0.6995   &  4.571  &   37.15 &   39.24
24-Qz &  7.6222 & 0.76490 &  15.8     & 0.9994   & 0.6989   &  4.574  &    0.29 &    0.3019
24+Qz &  16.796 & 0.76331 &  5.42     & 0.9972   [illegible]    1  &    1.21 &    0.9012
36-Qz &  13.902 & 0.76346 & 1.516E-02 &  1.000   [illegible]    0  &  305.4  &  321.0
36+Qz &  23.076 & 0.76346 & 2.537E-03 &  1.000   [illegible]    6  & 1906.   & 1938.
48-Qz &  20.182 & 0.76343 &  1.01     & 0.9992   [illegible]    1  &   11.24 &    4.875
```




which includes provision for comparison between MAD and output data.
Tunes and emittances are "measured" in the vicinity of the crossing gamma, namely, considering a few hundred turns around $(n\pm\nu_z)\,/\,G$.

This tool also produces a "control panel" :

# 4 Tracking through resonances

## 4.1 Intrinsic resonances

### 4.1.1 $\gamma G = \nu_z$ (3.648013 GeV)

#### Tracking data

```
Zgoubi.dat, excerpts :

Xing 0+nu
'OBJET'
14.12163e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   0.01 to 0.2e-6  =epsilon_z/pi
  0  1  0.
 'SCALING'
 1  2
  MULTIPOL SBEN
  -1
  14.12163
  1
  MULTIPOL QUAD
  -1
  14.12163
  1
 'PARTICUL'
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
.............
  'CAVITE'
  2.1
  807.043778118095   12.
  290.d3  0.5235987755982988731          9cavitiesx32kV, phi_s=30deg
   'MARKER'  #End
 'REBELOTE'
 4199  0.2 99
 'END'

------------------
From zgoubi.res :

     Particle  properties :
                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285
              Reference  data :
                    rigidity (kG.cm)      :   14121.6
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   4233.56
                    energy, total (MeV)   :   4336.29
                    energy, kinetic (MeV) :   3398.01
                    beta = v/c            :  0.9763099040
                    gamma                 :   4.621565007
                    beta*gamma            :   4.512079688
```

#### Strengths

From Figs. 10-23 one gets

| $\epsilon_z/\pi$ $(10^{-6})$ | $A^2$ | $\|J_n\|^2$ $(10^{-5})$ | $A^2/\epsilon_z/\pi$ | $\|J_n\|^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| 0.01 | 0.0357308 | 1.0030567 | 3573080 | 100.3057 | 1 | 0.9298 |
| 0.05 | 0.1788277 | 5.0201593 | 3576553 | 100.4032 | 1 | 0.6725 |
| 0.2 | 0.7467644 | 2.0963627 | 3733822. | 104.8181 | 0.9999 | -5.22E-02 |

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.01\,10^{-6}$

**Control screen**

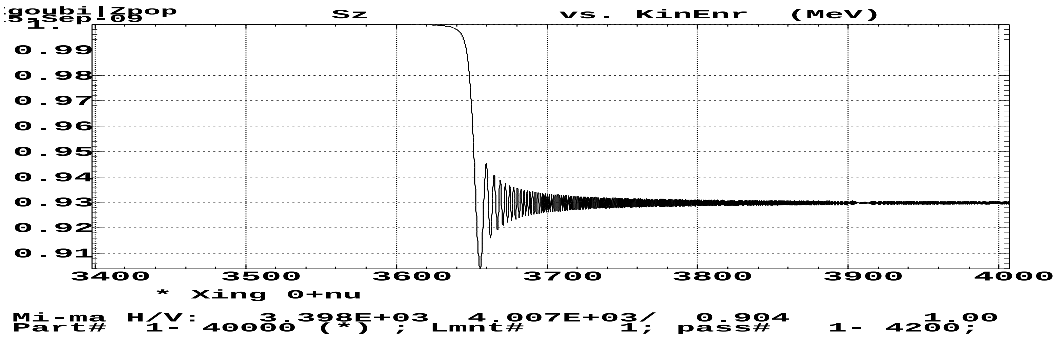



Figure 10: $S_z$ versus kinetic energy.

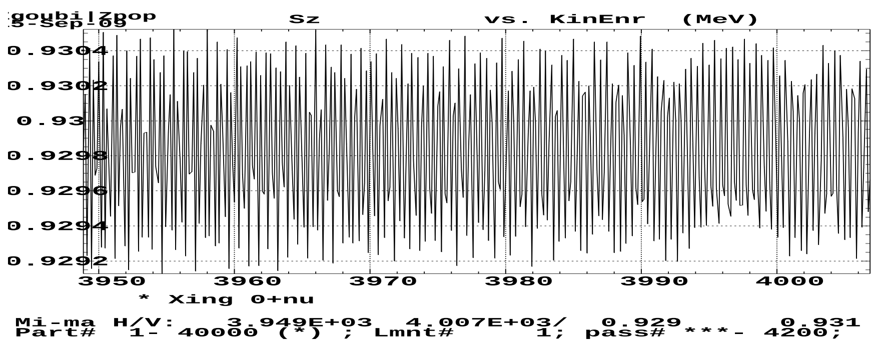



Figure 11: Zoom on final $S_z$.

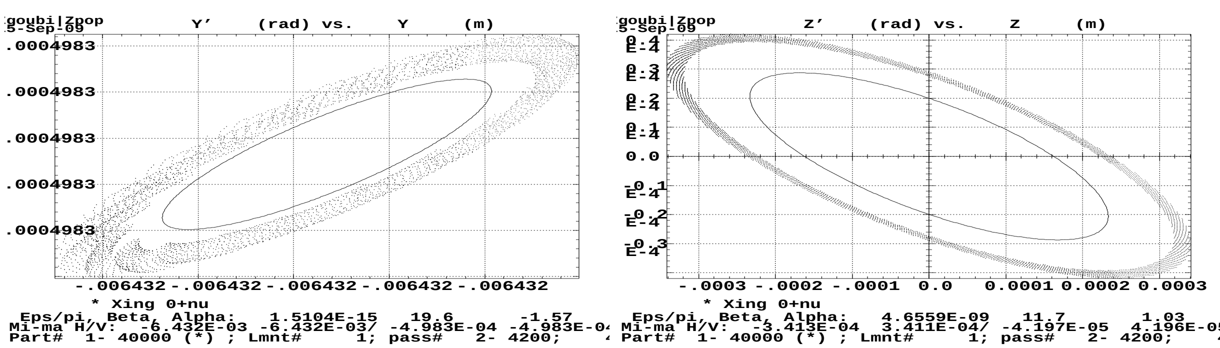



Figure 12: x-x' and z-z'.

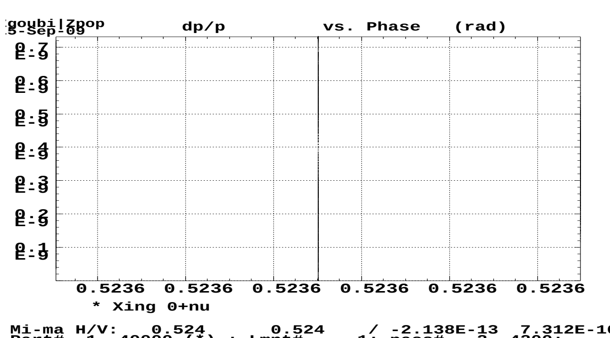



Figure 13: dp-phase.

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.05\,10^{-6}$

**Control screen**

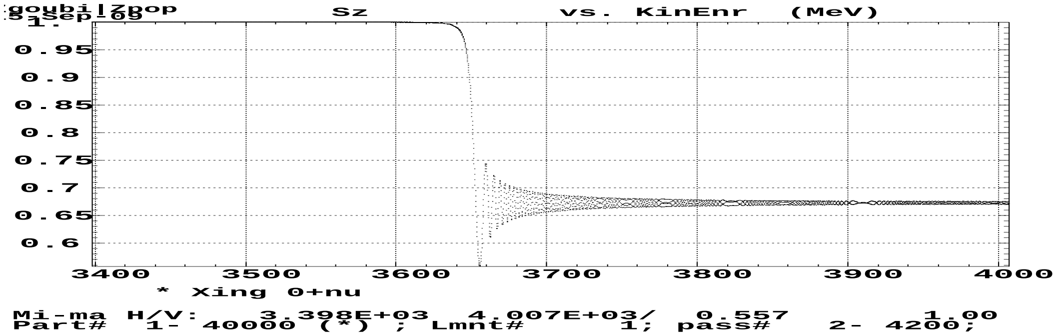



Figure 14: $S_z$ versus kinetic energy.

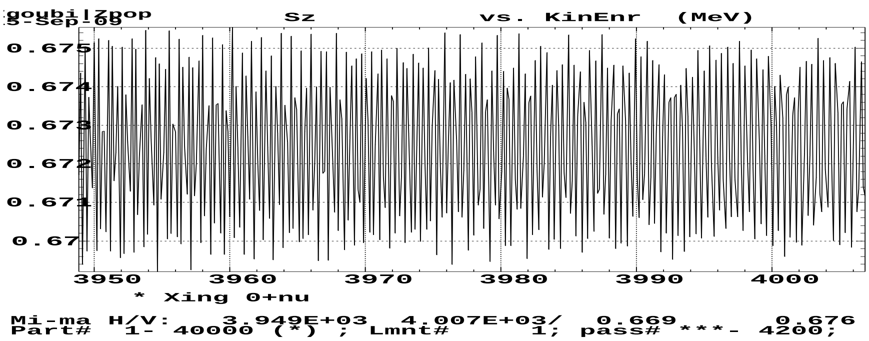



Figure 15: Zoom on final $S_z$.

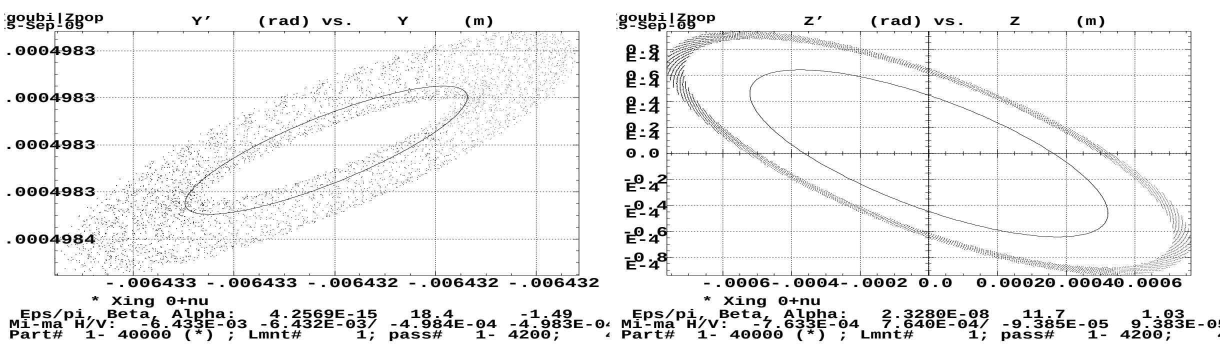



Figure 16: x-x' and z-z'.

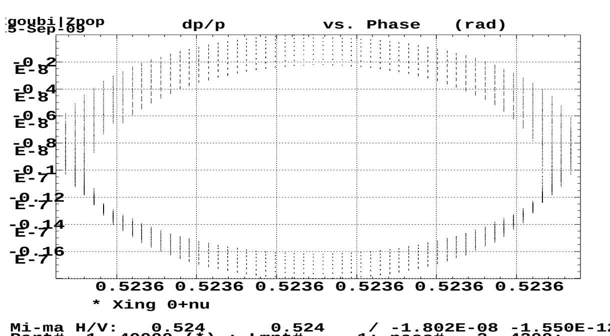



Figure 17: dp-phase.

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.2\,10^{-6}$

**Control screen**

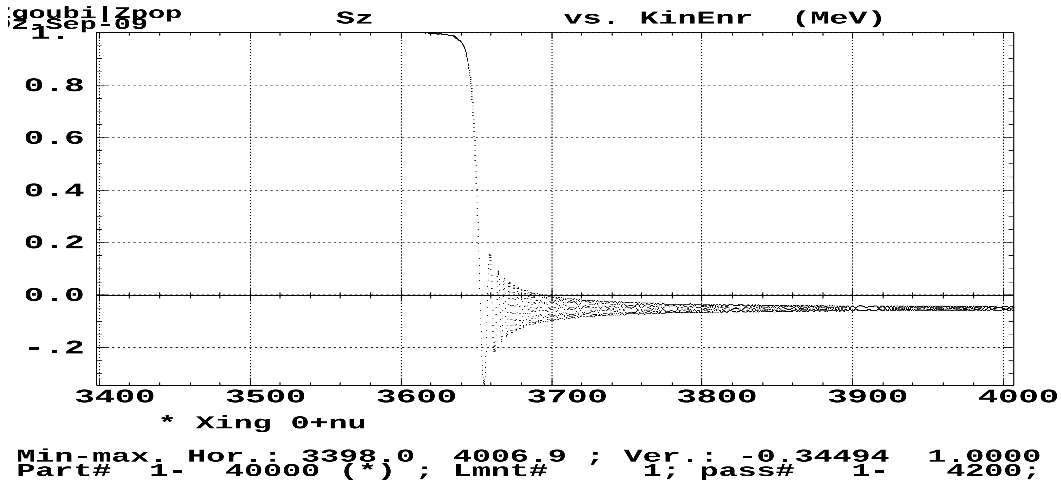



Figure 18: $S_z$ versus kinetic energy.

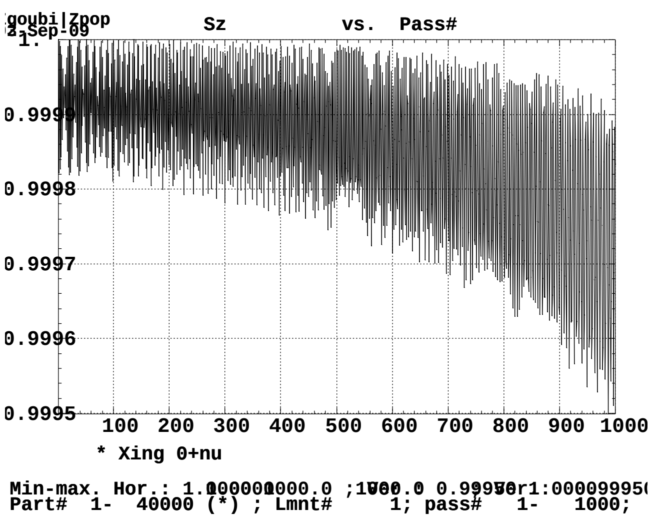



Figure 19: Zoom on initial $S_z$.

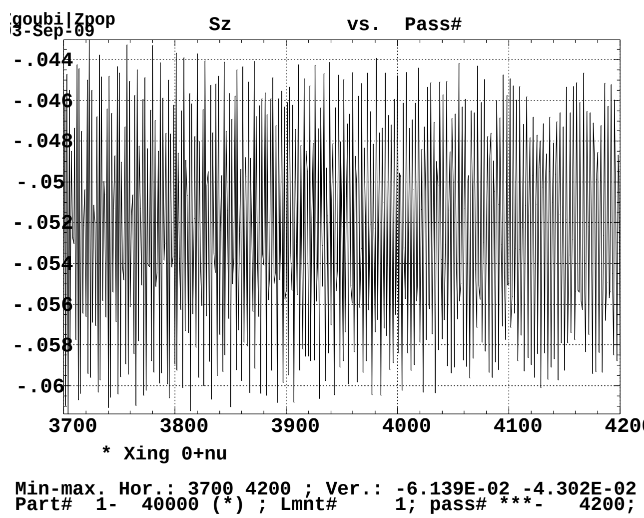



Figure 20: Zoom on final $S_z$.

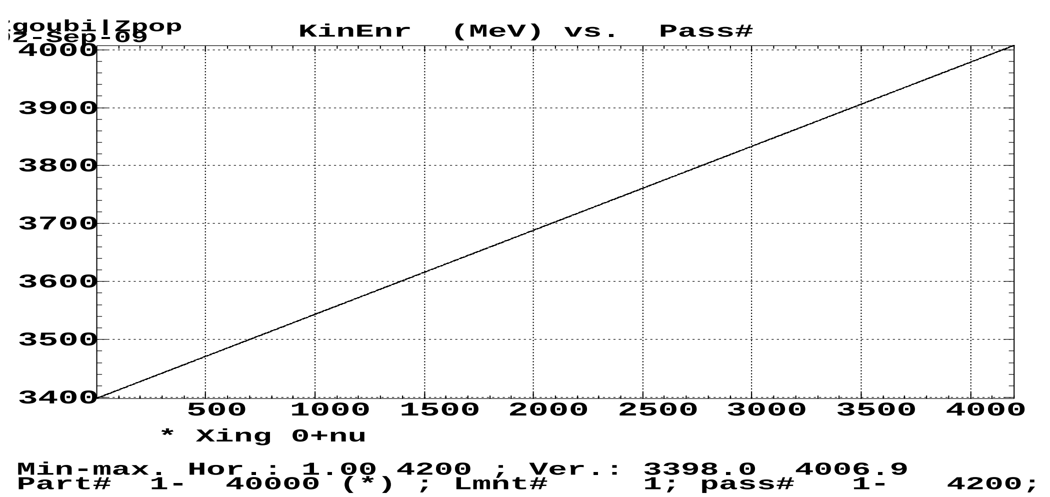



Figure 21: Kinetic E versus turn number.

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.2\,10^{-6}$

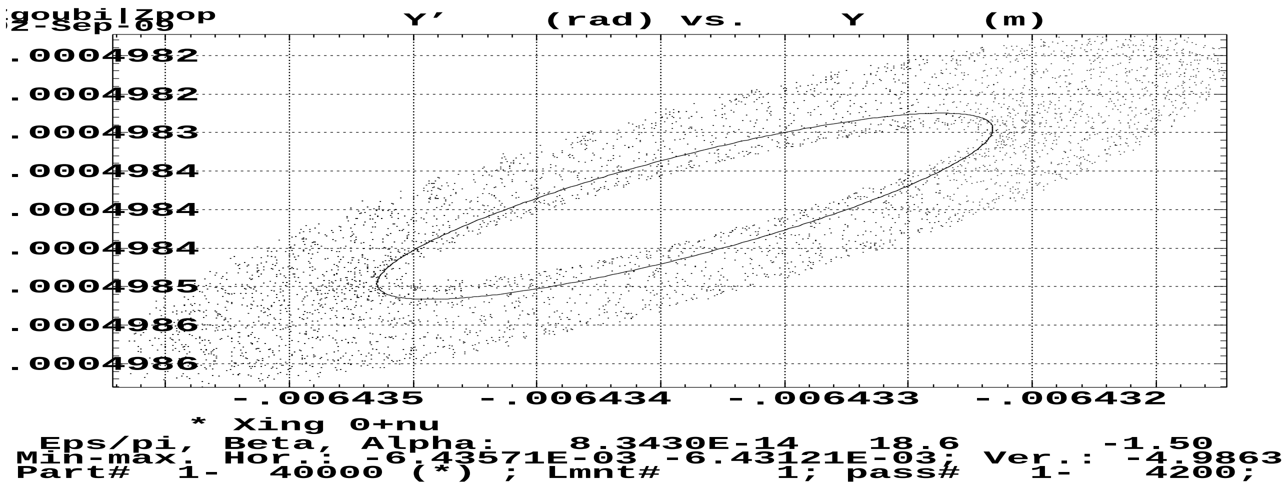



Figure 22: x-x'.

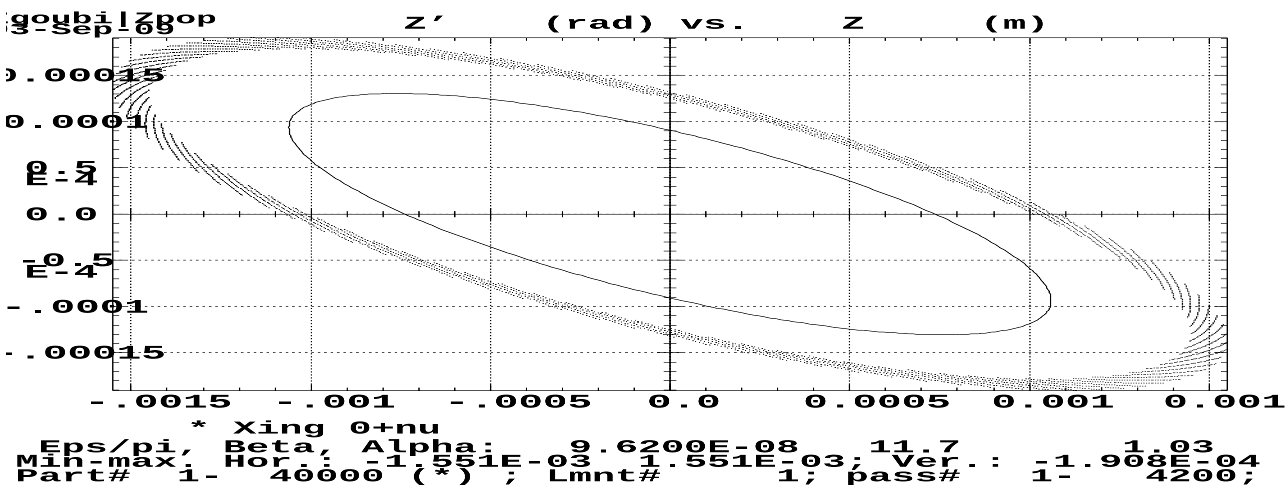



Figure 23: z-z'.

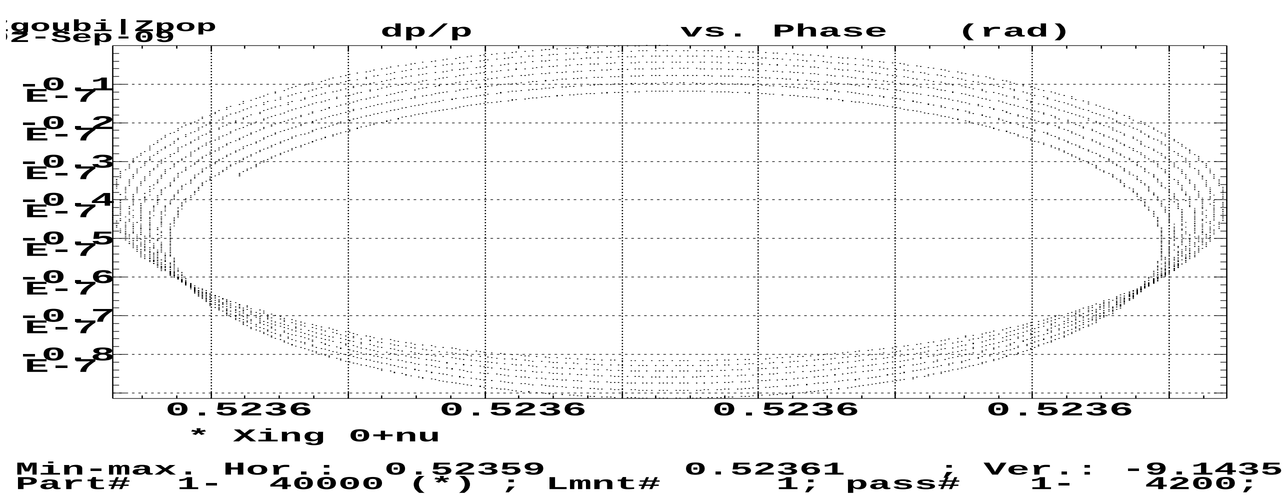



Figure 24: dp-phase.

### Testing possible effect of betatron phase

7 particles evenly spread on $\epsilon_z/\pi = 0.2$ mm.mrad invariant are launched : no observable difference in spin dynamics.

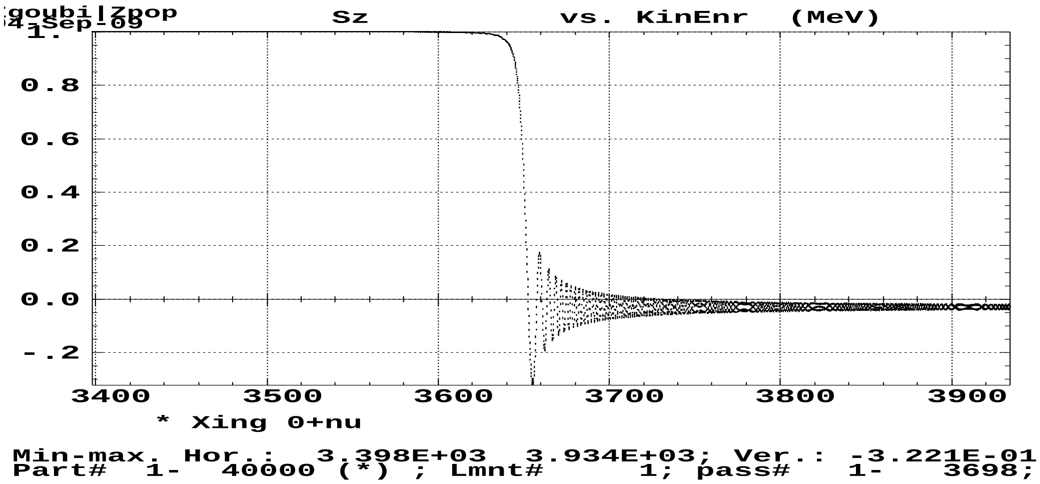



Figure 25: $S_z$ versus kinetic energy, 7 particles evenly spread on $\epsilon_z/\pi = 0.2$ mm.mrad invariant.

### 4.1.2 $\gamma G = 24 - \nu_z$ (7.82892 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

Xing 24-nu. Data generated by  geneZGDat4Xing
   'OBJET'
   25.92623852d3
    8
     1  1  1
    -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
    -1.588  19.843  0.
    1.033  11.675   .1e-6
     0  1  0.
 'PARTICUL'
  9.382720300E+02  1.602176487E-19  1.792847400E+00  0. 0.
 'SPNTRK'
   3
 'FAISCEAU'
 'SCALING'
  1  2
   MULTIPOL SBEN
   -1
   25.92623852
   1
   MULTIPOL QUAD
   -1
   25.92623852
   1
 'PICKUPS'
    1
    #Start
 'SPNSTORE'
    b_zgoubi.spn  #Start
      1
 'FAISTORE'
    b_zgoubi.fai  #End
      1
 'MARKER'  #Start
 'MARKER'   MARK      BSUPERA
 'MULTIPOL' SBEN      A1BF
 0  .Dip
 200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
......................
......................
 'CAVITE'
  2   .1
   807.04377811810     12.00
  2.90000000E+05      2.617993877991E+00
 'MARKER'   #End
 'REBELOTE'
    2500  0.2  99
```

## Strengths

| $\epsilon_z/\pi$ ($10^{-6}$) | $A^2$ | $\lvert J_n\rvert^2$ ($10^{-6}$) | $A^2/\epsilon_z/\pi$ | $\lvert J_n\rvert^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| 0.1 | 1.050547E-03 | 2.94915967E-2 | 10505 | 0.29491597 | 1. | 0.997900 |

$\gamma G = 24 - \nu_z$ (7.82892 GeV) - $\epsilon_z/\pi = 0.1\,10^{-6}$

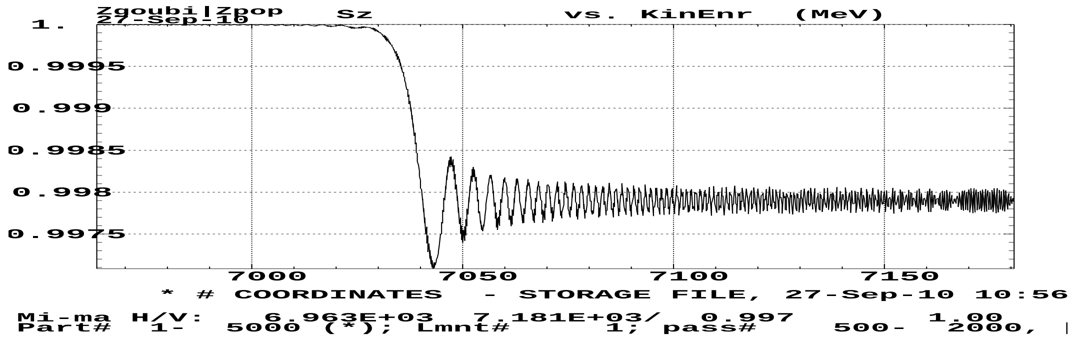



Figure 26: $S_z$ versus kinetic energy.

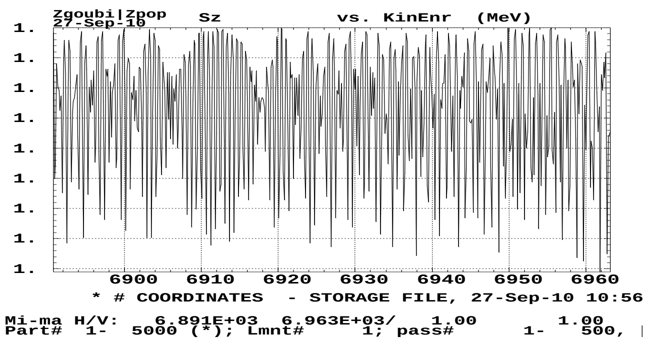



Figure 27: Zoom on initial $S_z$.

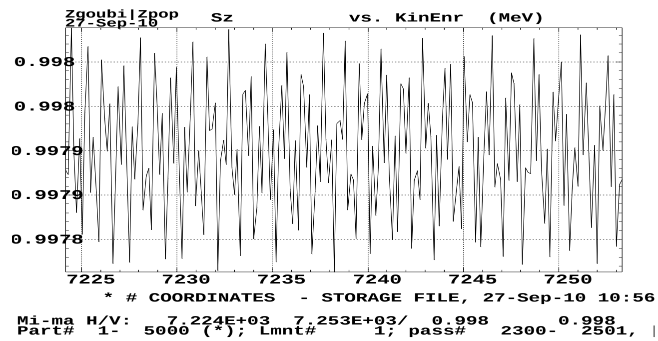



Figure 28: Zoom on final $S_z$.

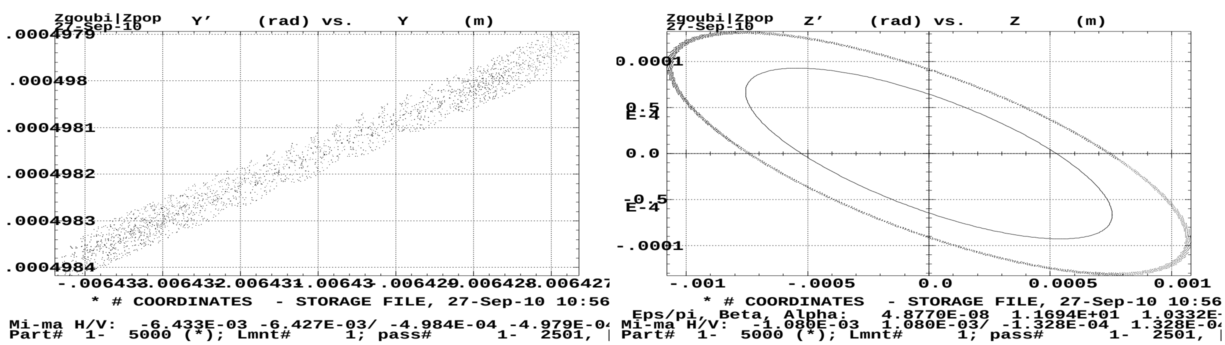



Figure 29: x-x' and z-z'.

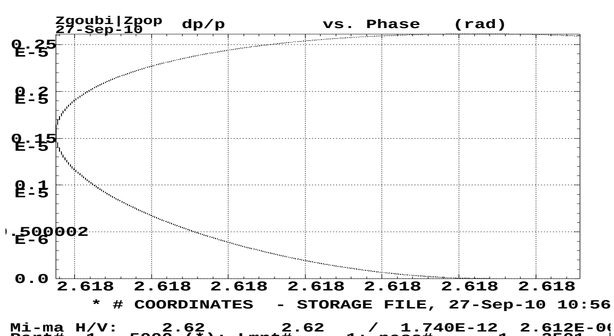



Figure 30: dp-phase.

### 4.1.3 $\gamma G = 12 + \nu_z$ (7.82892 GeV)

#### Tracking data

```
Zgoubi.dat, excerpts :

Xing 12+nu. Data generated by  geneZGDat4Xing
      'OBJET'
   35.38271465d3
    8
        1  1  1
       -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
       -1.588  19.843  0.
       1.033  11.675   .1e-6
        0  1  0.
 'PARTICUL'
  9.382720300E+02  1.602176487E-19  1.792847400E+00  0. 0.
 'SPNTRK'
   3
 'FAISCEAU'
 'SCALING'
  1  2
   MULTIPOL SBEN
   -1
   35.38271465
   1
   MULTIPOL QUAD
   -1
   35.38271465
   1
 'PICKUPS'
    1
    #Start
 'SPNSTORE'
    b_zgoubi.spn  #Start
      1
 'FAISTORE'
    b_zgoubi.fai  #End
      1
 'MARKER'  #Start
 'MARKER'   MARK      BSUPERA
 'MULTIPOL' SBEN      A1BF
 0  .Dip
 200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|200|20    Dip  A1BF
 3   0.00000000000000        0.00000000000000      -1.175115045000000E-002
......................
......................
 'CAVITE'
  2   .1
   807.04377811810     12.00
  2.90000000E+05     2.617993877991E+00
 'MARKER'   #End
 'REBELOTE'
    3000  0.2  99
```

#### Strengths

| $\epsilon_z/\pi$ $(10^{-6})$ | $A^2$ | $\|J_n\|^2$ $(10^{-6})$ | $A^2/\epsilon_z/\pi$ | $\|J_n\|^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| 0.002 | 0.00256527 | 0.07201394 | 1282637 | 36.006969 | 1. | 0.994876 |
| 0.1 | 0.12897037 | 3.62053515 | 1289703 | 36.205349 | 1. | 0.7590 |
| 2 | 2.5431926 | 71.3940608 | 1271596 | 35.697029 | 1. | -0.842770 |

$\gamma G = 12 + \nu_z$ (7.82892 GeV) - $\epsilon_z/\pi = 0.002\,10^{-6}$

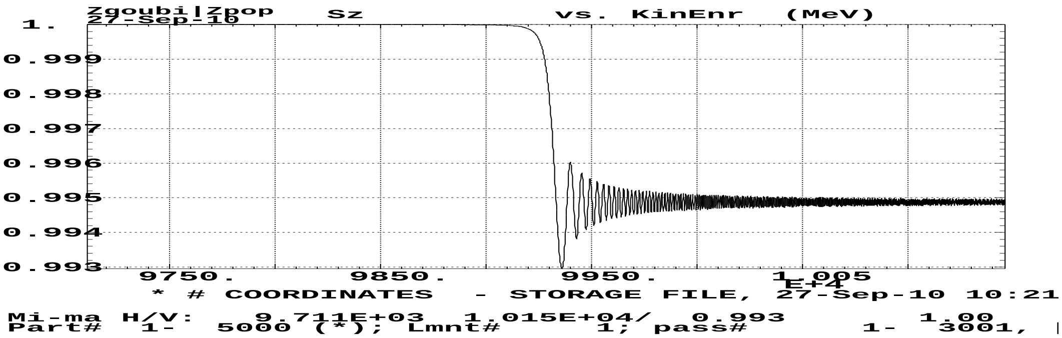



Figure 31: $S_z$ versus kinetic energy.

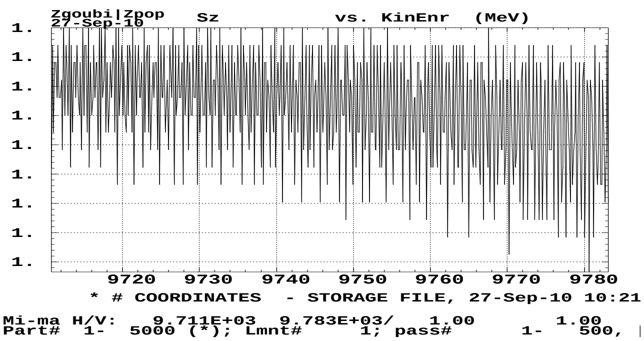



Figure 32: Zoom on initial $S_z$.

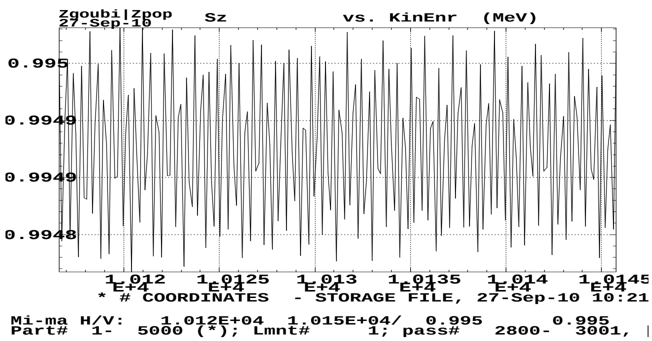



Figure 33: Zoom on final $S_z$.

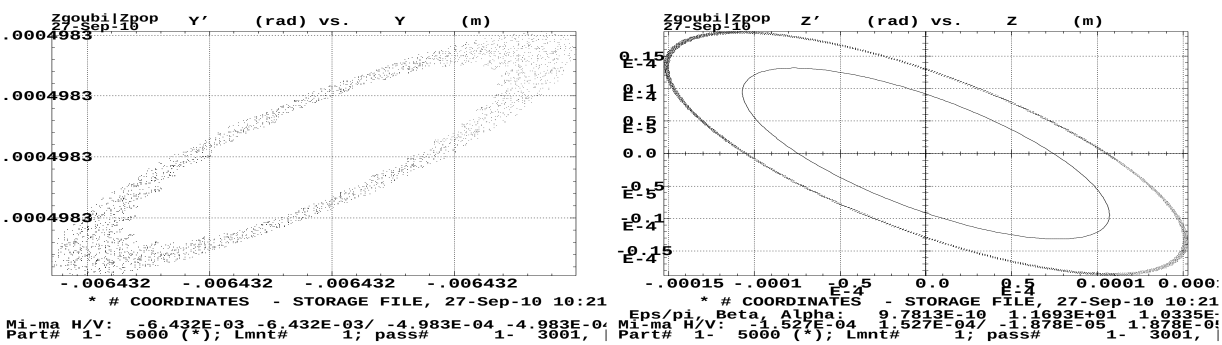



Figure 34: x-x' and z-z'.

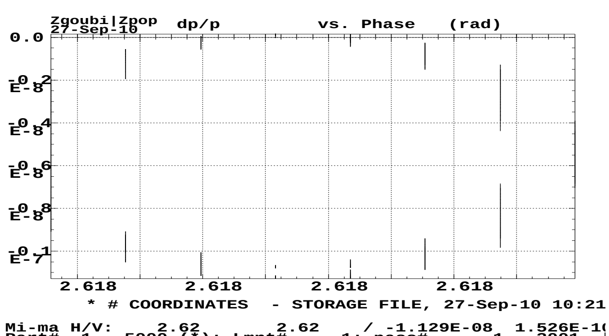



Figure 35: dp-phase.

$\gamma G = 12 + \nu_z$ (7.82892 GeV) - $\epsilon_z/\pi = 2\,10^{-6}$

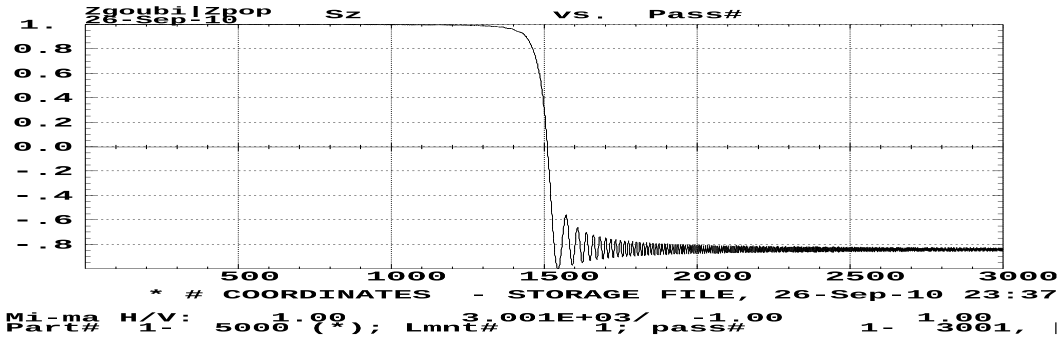



Figure 36: $S_z$ versus kinetic energy.

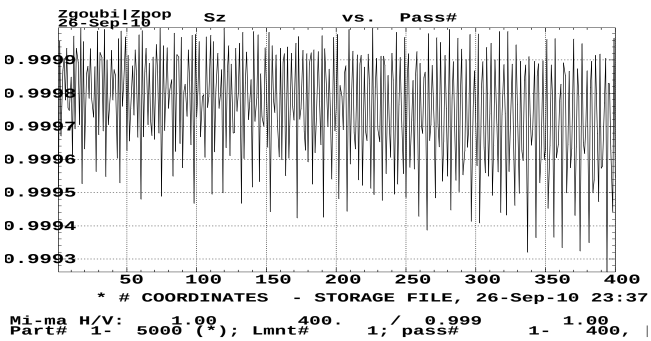



Figure 37: Zoom on initial $S_z$.

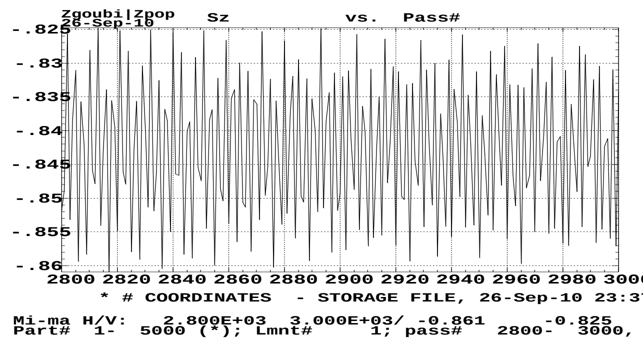



Figure 38: Zoom on final $S_z$.

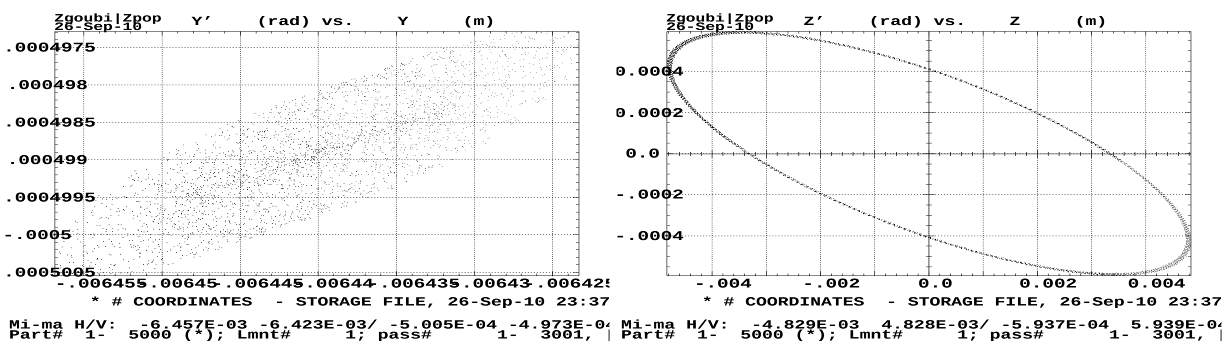



Figure 39: x-x' and z-z'.

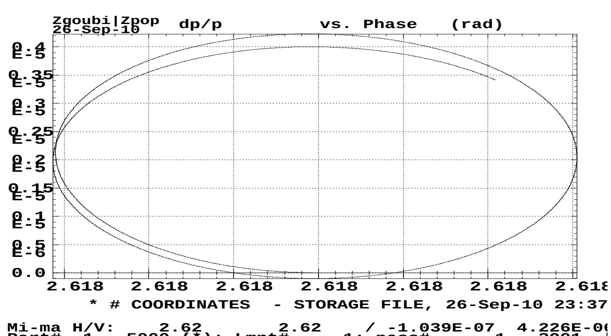



Figure 40: dp-phase.

### 4.1.4 $\gamma G = 23 + \nu_z$ (15.68487 GeV)

#### Tracking data

```
Zgoubi.dat, excerpts :

Xing 23+nu
'OBJET'
54.92333e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   10e-4
  0  1  0.
'FAISCEAU'
 'SCALING'                                                                                                                  3
 1  2
  MULTIPOL SBEN
  -1
  54.92333
  1
  MULTIPOL QUAD
  -1
  54.92333
  1
```

#### Strength

No visible resonance, $S_z \simeq 1,\ \forall\, G\gamma$, up to unreasonable V emittances...

### 4.1.5 $\gamma G = 24 + \nu_z$ (16.20822 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

Xing 24+nu
'OBJET'
56.67175e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   30e-6  to
  0  1  0.
'FAISCEAU'
 'SCALING'                                                                                                                  3
 1  2
  MULTIPOL SBEN
  -1
  56.67175
  1
  MULTIPOL QUAD
  -1
  56.67175
  1
......................
......................
  'CAVITE'                                                              85                                        86
  2.1         .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365       9cavitiesx32kV, phi_s=180-30deg
```

## Strengths

| $\epsilon_z/\pi$ $(10^{-6})$ | $A^2$ | $\lvert J_n\rvert^2$ $(10^{-5})$ | $A^2/\epsilon_z/\pi$ | $\lvert J_n\rvert^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| 30 | 1.011348 | 2.83912 | 33711.59 | 0.9463723 | 0.987 | -0.2690 |
| 2 | 0.0737230 | 0.20696 | 36861.53 | 1.034799 | 0.999 | 0.857 |

$\gamma G = 24 + \nu_z$ (16.20822 GeV) - $\epsilon_z/\pi = 2\,10^{-6}$

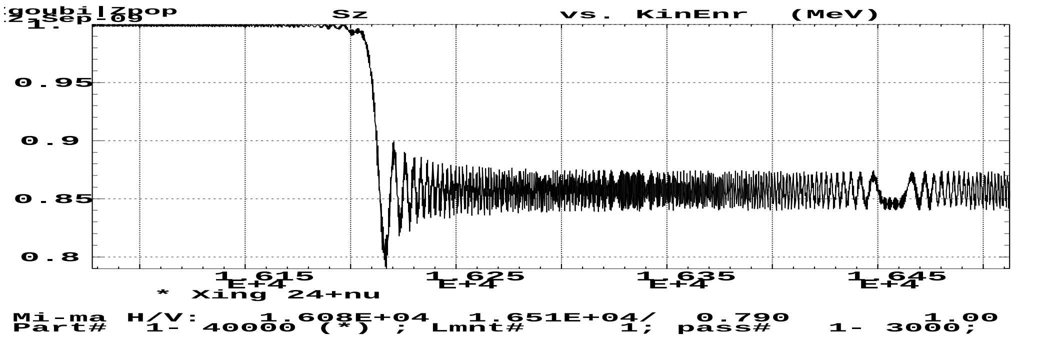



Figure 41: $S_z$ versus kinetic energy.

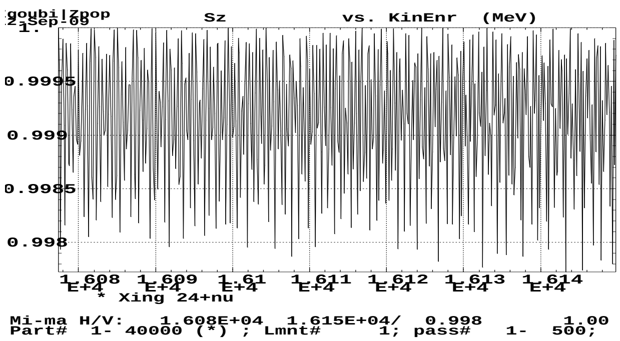



Figure 42: Zoom on initial $S_z$.

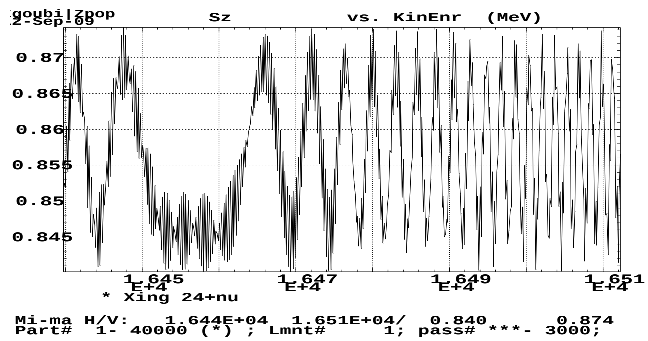



Figure 43: Zoom on final $S_z$.

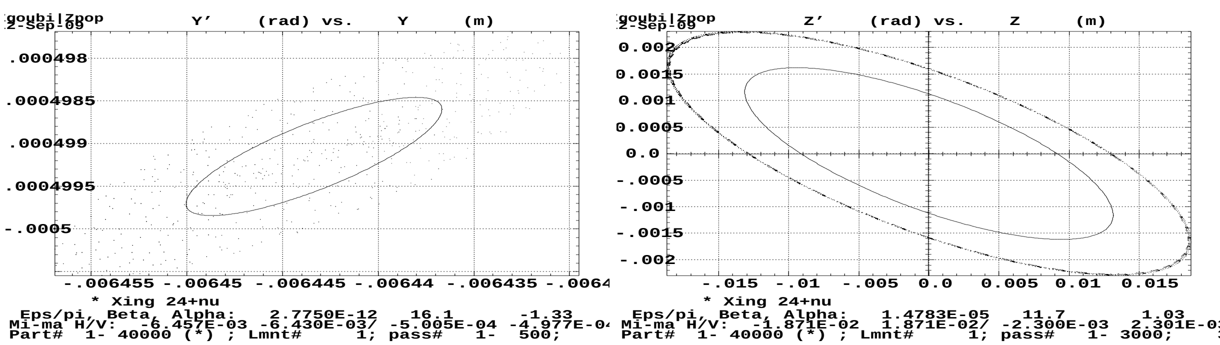



Figure 44: x-x' and z-z'.

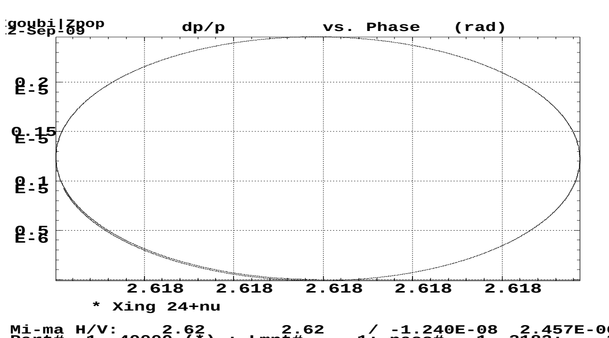



Figure 45: dp-phase.

$\gamma G = 24 + \nu_z$ (16.20822 GeV) - $\epsilon_z/\pi = 30\,10^{-6}$

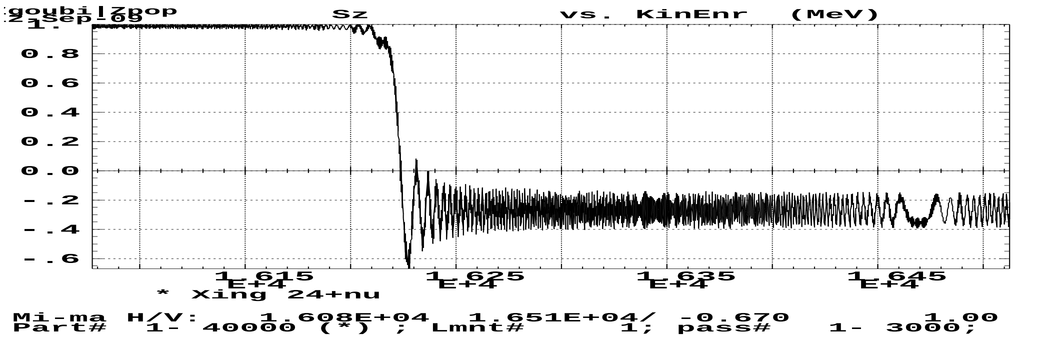



Figure 46: $S_z$ versus kinetic energy.

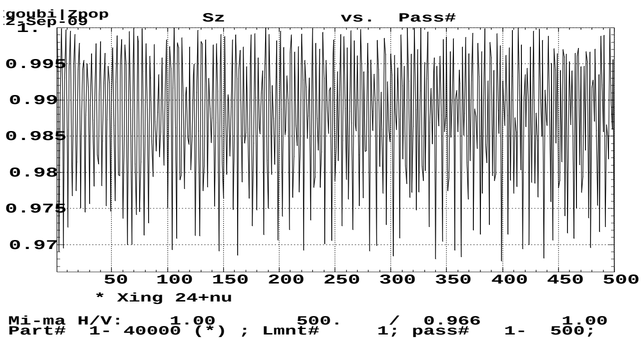



Figure 47: Zoom on initial $S_z$.

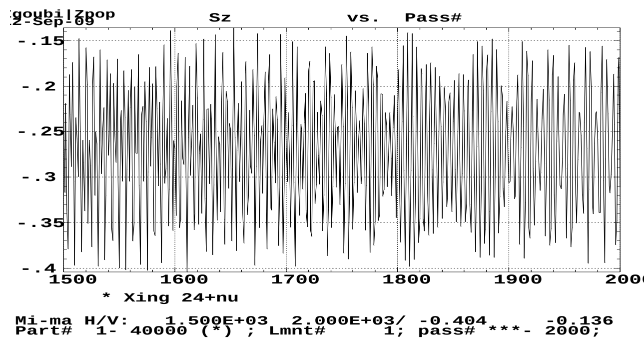



Figure 48: Zoom on final $S_z$.

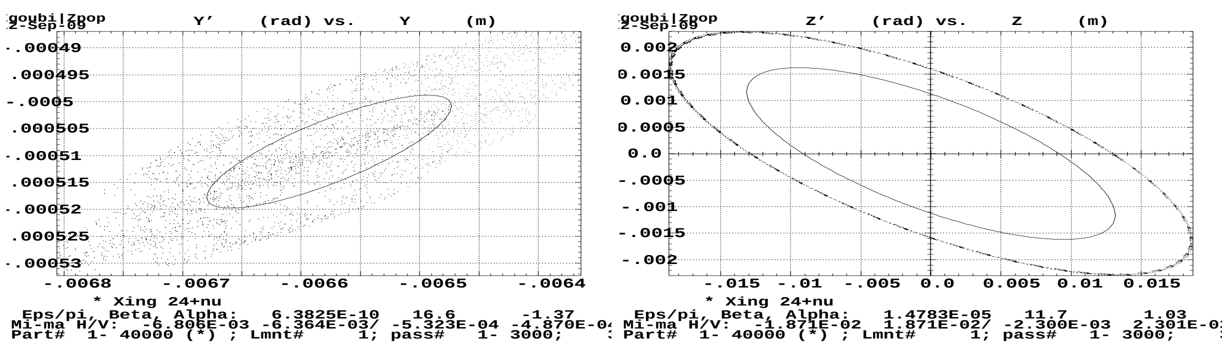



Figure 49: x-x' and z-z'.

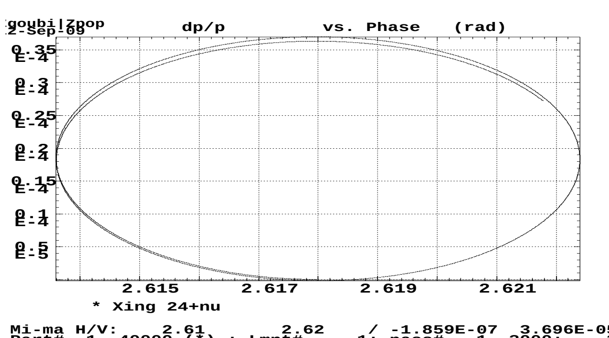



Figure 50: dp-phase.

### 4.1.6 $\gamma G = 36 - \nu_z$ (13.31575 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

Xing 36-nu
'OBJET'
47.00580e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   .5e-7 =epsilon_z/pi
  0  1  0.
 'SCALING'                                                                                                                  3
 1  2
  MULTIPOL SBEN
  -1
  47.00580
  1
  MULTIPOL QUAD
  -1
  47.00580
  1
 'PARTICUL'                                                                                                                 4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
.............
  'CAVITE'                                                                  85                                              86
  2.1         .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365       9cavitiesx32kV, phi_s=180-30deg
------------------
From zgoubi.res :

     Particle  properties :
                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285

              Reference  data :
                    rigidity (kG.cm)      :   47005.8
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   14092.0
                    energy, total (MeV)   :   14123.2
                    energy, kinetic (MeV) :   13184.9
                    beta = v/c            :  0.9977907636
                    gamma                 :   15.05233592
                    beta*gamma            :   15.01908175
```

## Strength

From Figs. 52, 53 one gets

$p_{init} \approx 0.9998300, \quad p_{final} \approx 0.1545000$

Eq. 2 yields

$A^2 = 0.5494571$

$|J_n|^2 = 1.5424697E-05$

$\gamma G = 36 - \nu_z$ **(13.31575 GeV)**

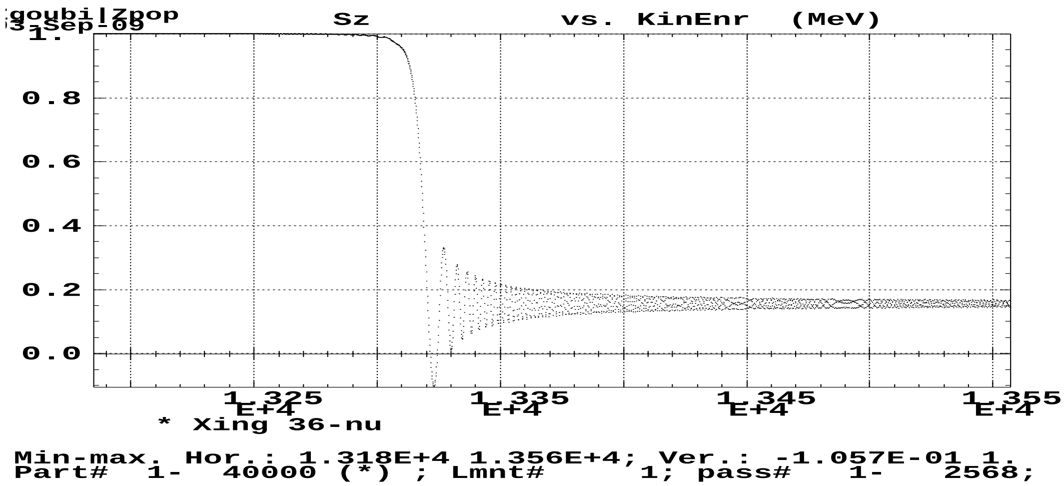



Figure 51: $S_z$ versus kinetic energy.

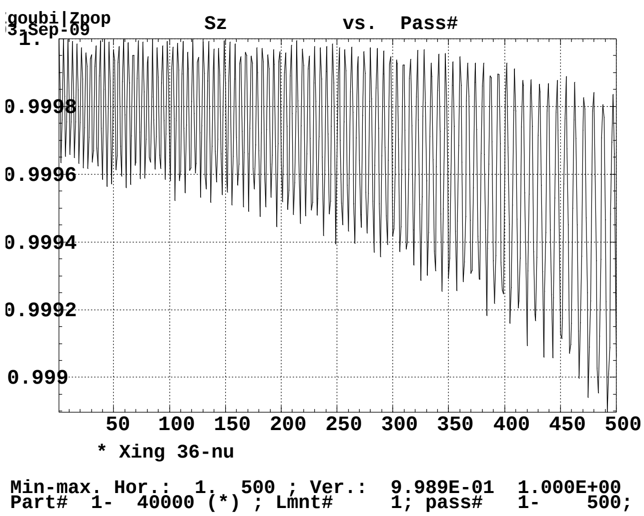



Figure 52: Zoom on initial $S_z$.

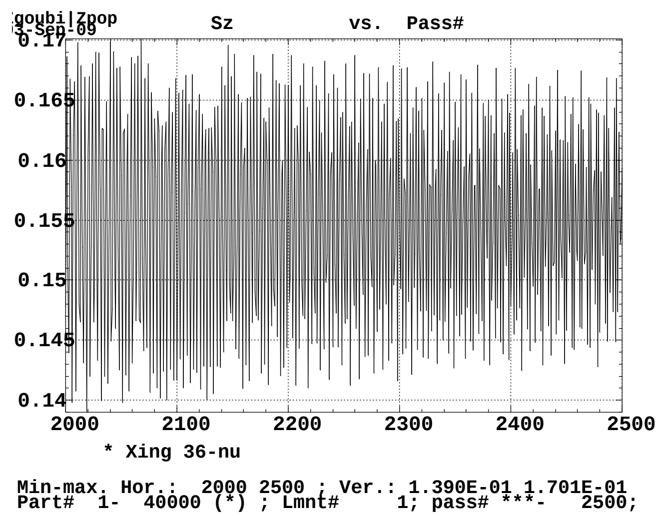



Figure 53: Zoom on final $S_z$.

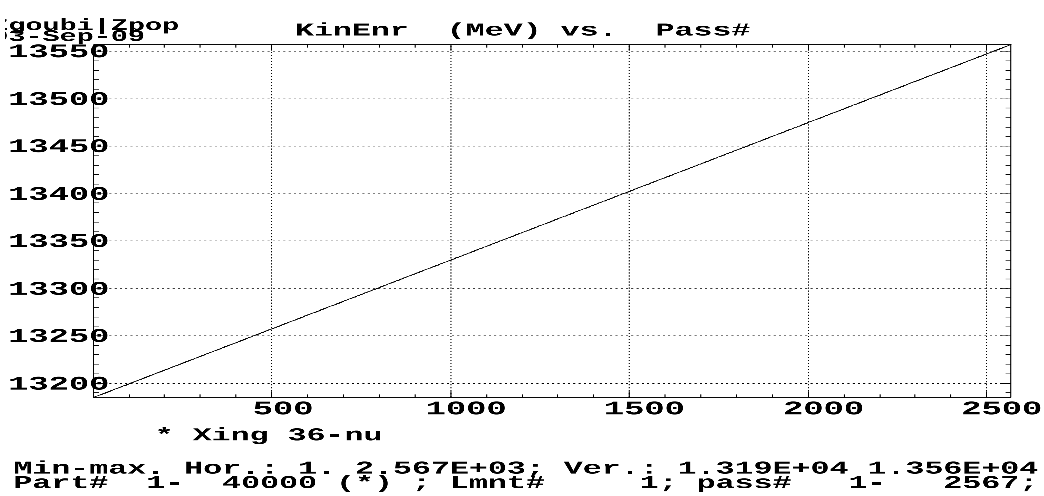



Figure 54: Kinetic E versus turn number.

$\gamma G = 36 - \nu_z$ **(13.31575 GeV)**

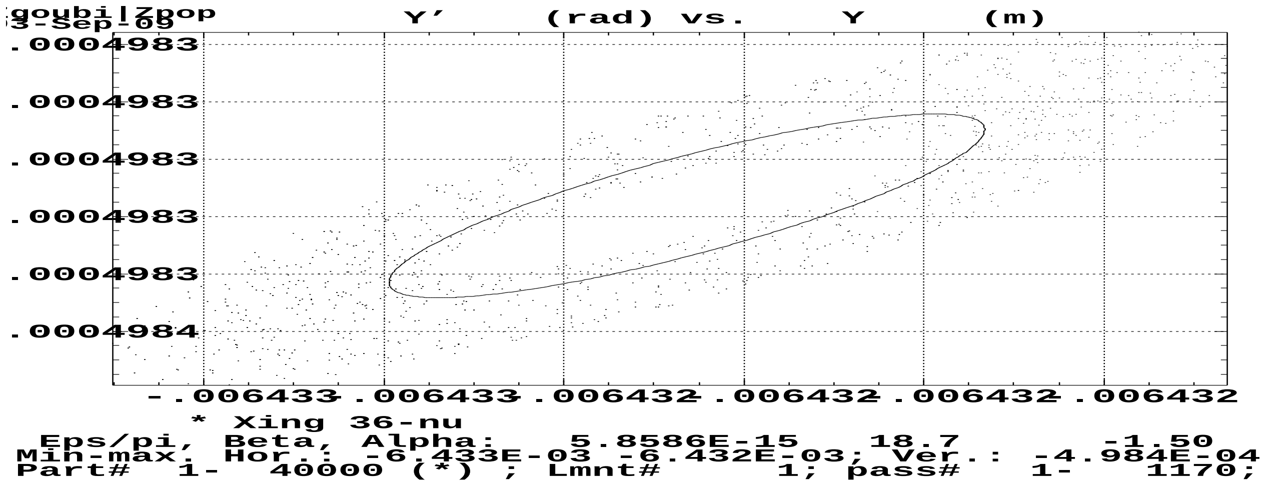



Figure 55: x-x'.

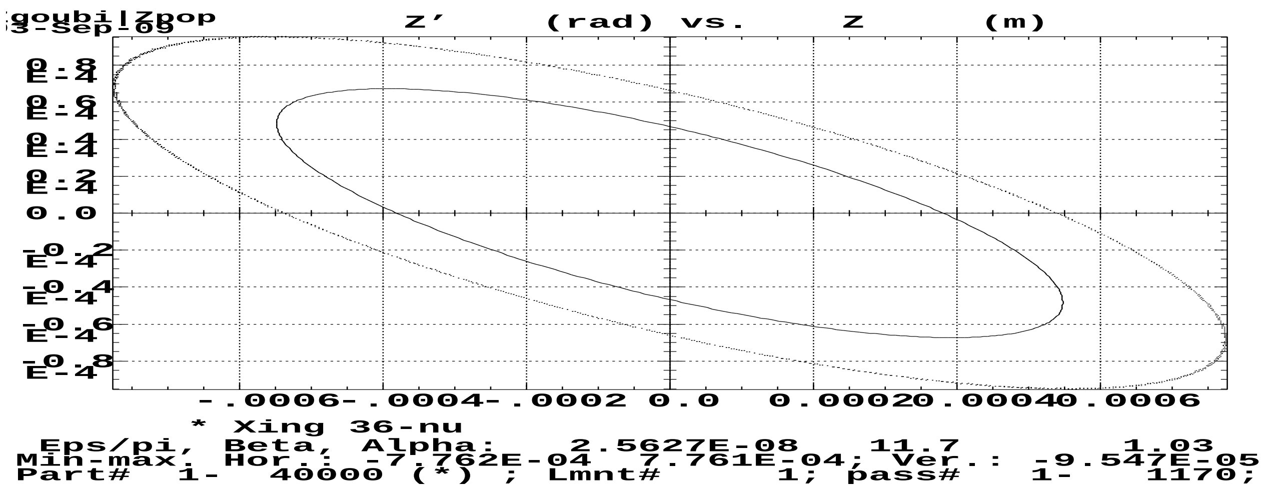



Figure 56: z-z'.

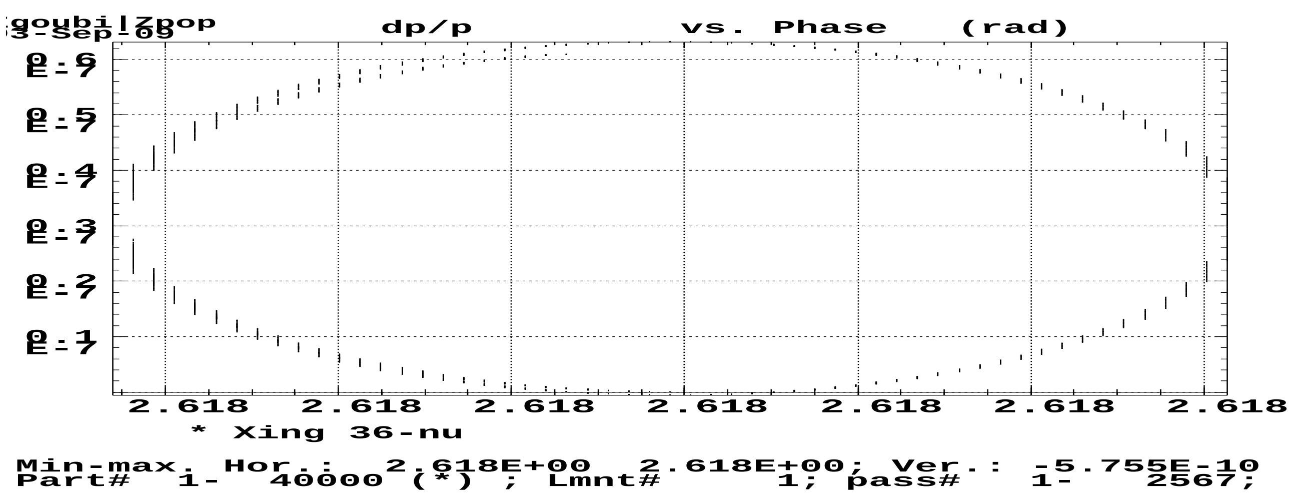



Figure 57: dp-phase.

### 4.1.7 $\gamma G = 48 - \nu_z$ (19.59585 GeV)

**Tracking data**

```
Zgoubi.dat, excerpts :

Xing 48-nu
'OBJET'
67.98605e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675     0.5 to 2e-6
  0  1  0.
 'SCALING'                                                                                                                   3
 1  2
  MULTIPOL SBEN
  -1
  67.98605
  1
  MULTIPOL QUAD
  -1
  67.98605
  1
```

**Strength**

Eq. 2 yields the results displayed in Tab. 4.

Table 4: Resonance strength for various vertical invariant values.

| $\epsilon_z/\pi$ ($\times 10^{-6}$) | $A^2$ | $\|J_n\|^2$ | $A^2/\epsilon_z/\pi$ | $\|J_n\|^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| .125 | 0.04630575 | 1.2999233E-06 | 370446.0 | 10.39939 | 1 | 0.9095 |
| .25 | 0.09862635 | 2.7686992E-06 | 394505.4 | 11.07480 | 0.9998 | 0.812 |
| .5 | 0.1913054 | 5.3704412E-06 | 382610.8 | 10.74088 | 0.9996 | 0.6515 |
| 1 | 0.3895315 | 1.0935165E-05 | 389531.5 | 10.93517 | 0.9993 | 0.3545 |
| 2 | 0.8073937 | 2.2665648E-05 | 403696.8 | 11.33282 | 0.9985 | -0.1078 |

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.125\,10^{-6}$

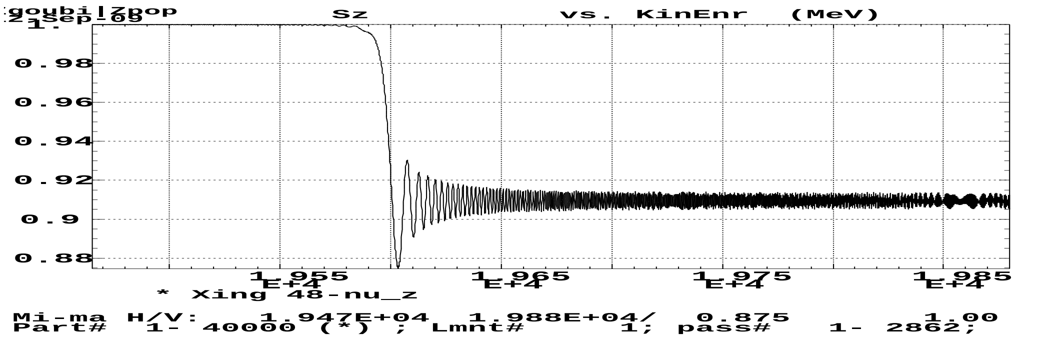



Figure 58: $S_z$ versus kinetic energy.

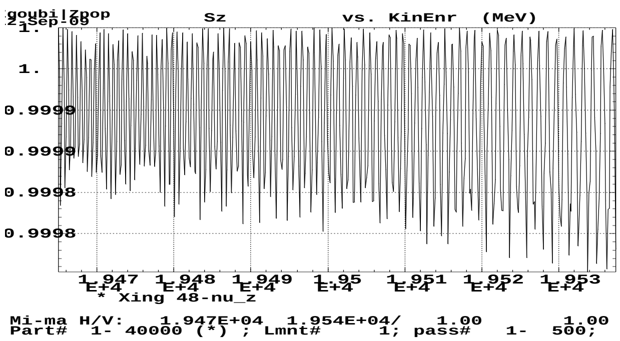



Figure 59: Zoom on initial $S_z$.

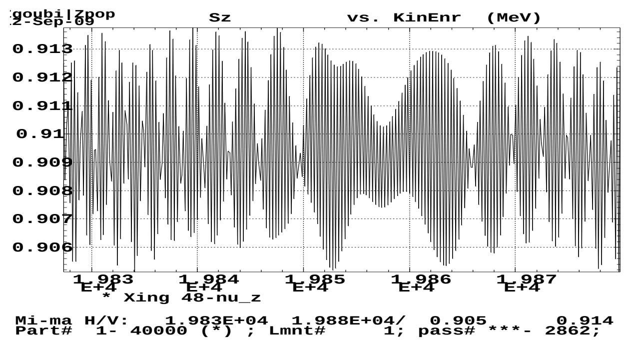



Figure 60: Zoom on final $S_z$.

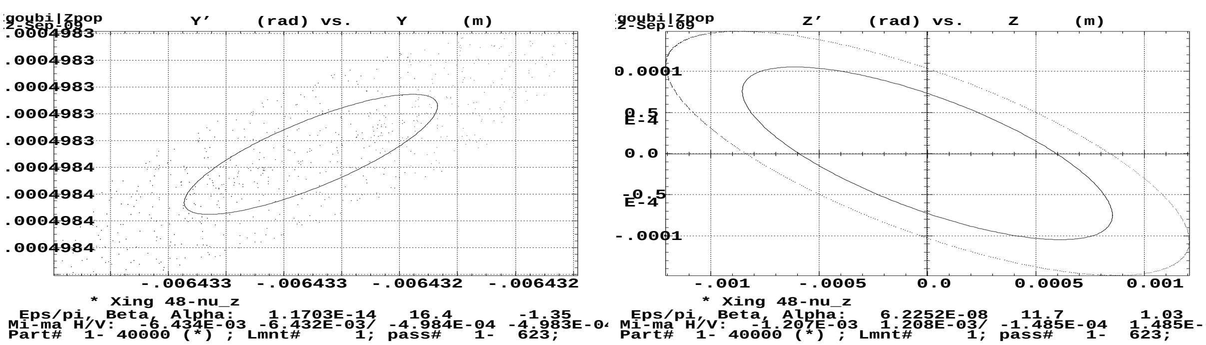



Figure 61: x-x' and z-z'.

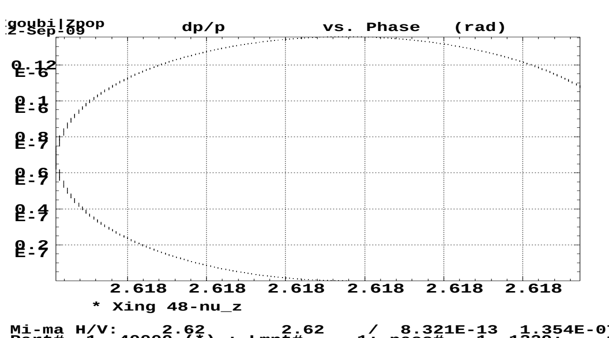



Figure 62: dp-phase (1350 turns).

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.25\,10^{-6}$

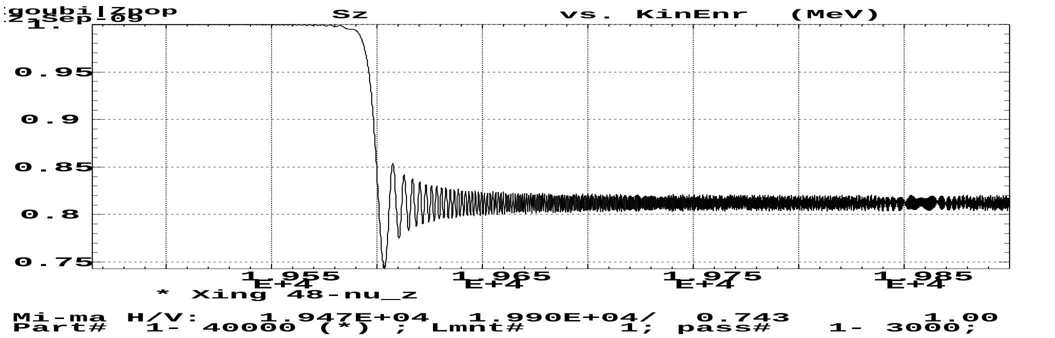



Figure 63: $S_z$ versus kinetic energy.

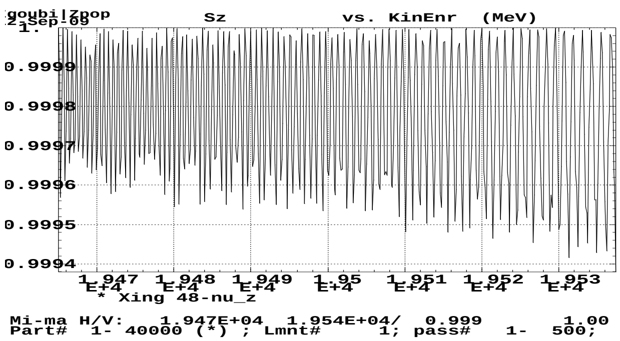



Figure 64: Zoom on initial $S_z$.

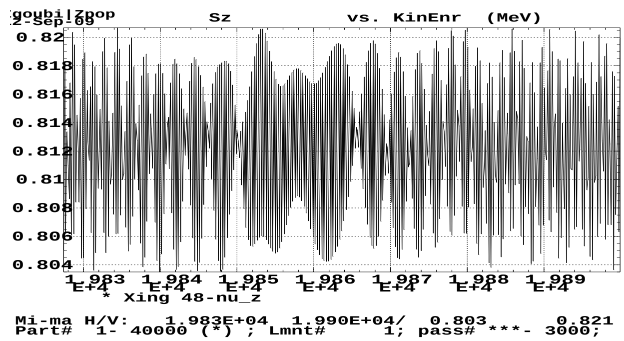



Figure 65: Zoom on final $S_z$.

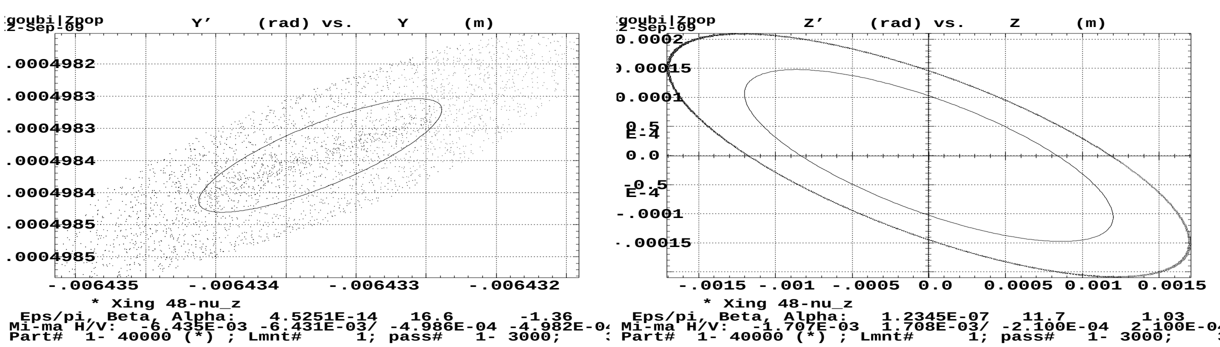



Figure 66: x-x' and z-z'.

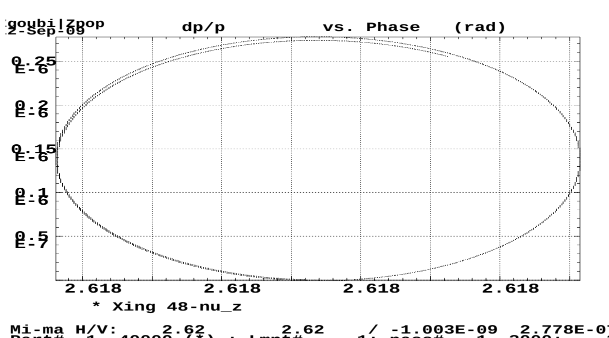



Figure 67: dp-phase.

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.5\,10^{-6}$

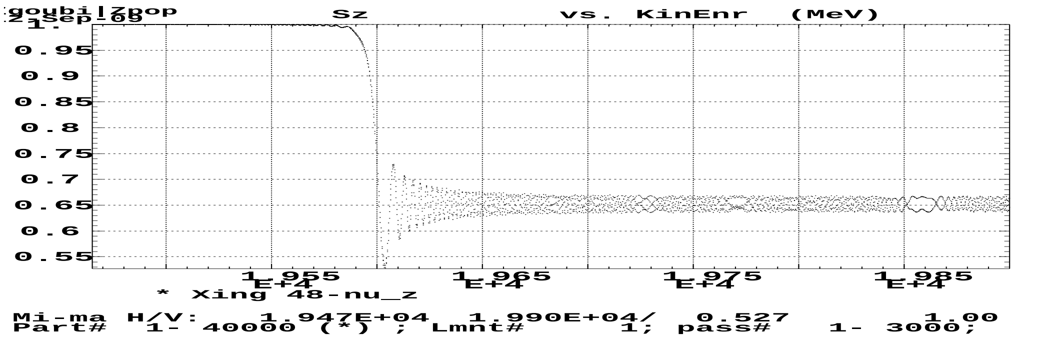



Figure 68: $S_z$ versus kinetic energy.

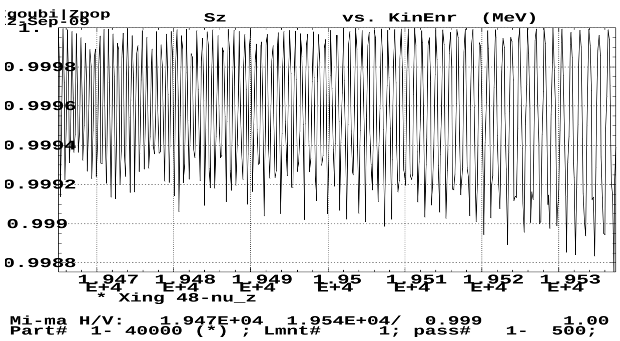



Figure 69: Zoom on initial $S_z$.

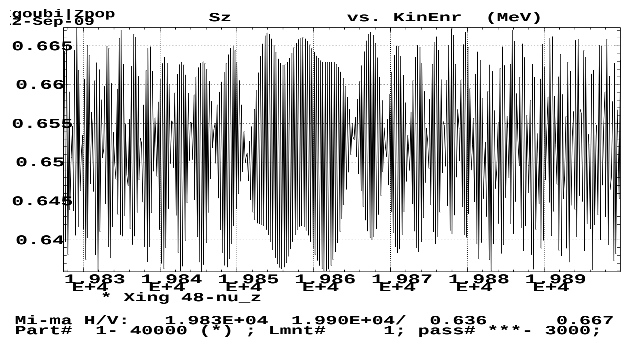



Figure 70: Zoom on final $S_z$.

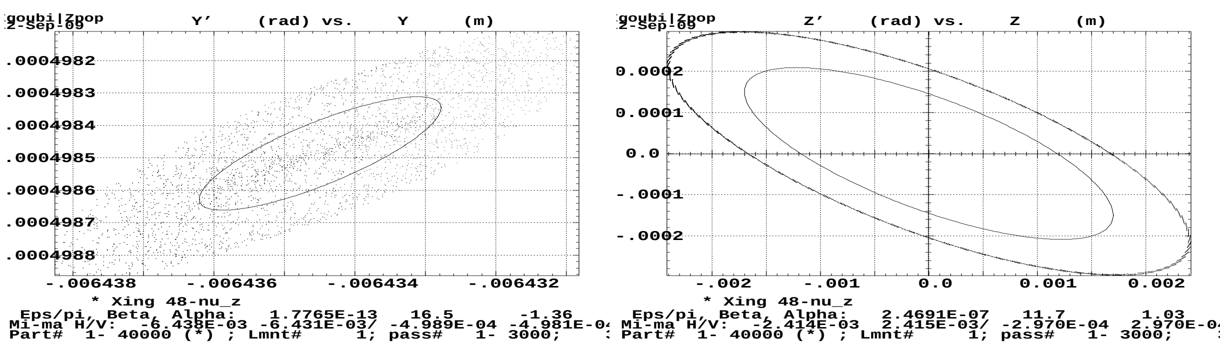



Figure 71: x-x' and z-z'.

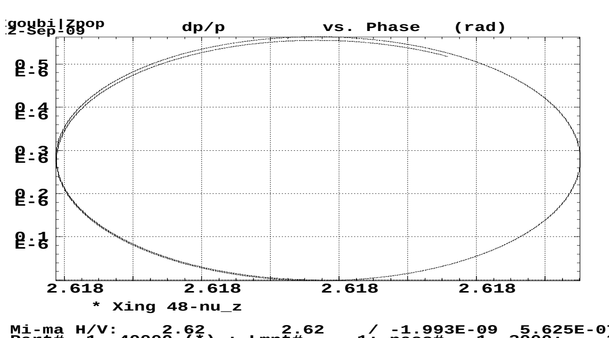



Figure 72: dp-phase.

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 10^{-6}$

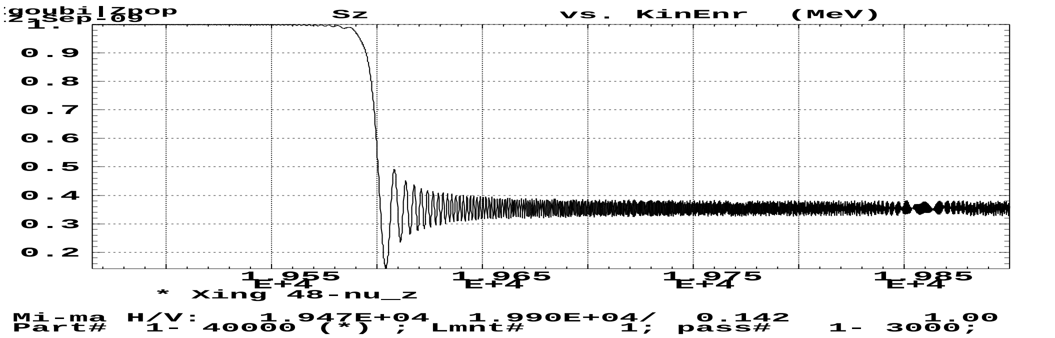



Figure 73: $S_z$ versus kinetic energy.

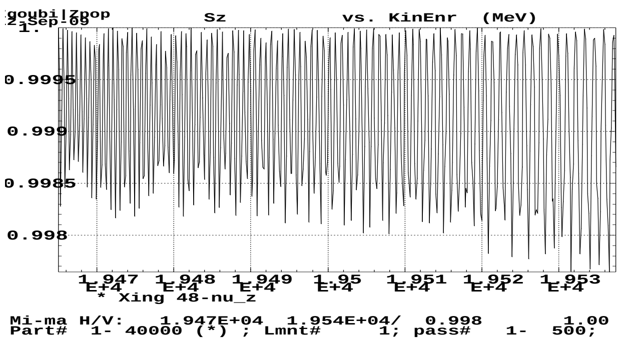



Figure 74: Zoom on initial $S_z$.

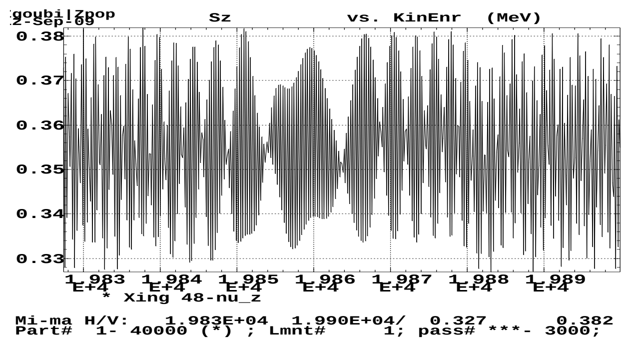



Figure 75: Zoom on final $S_z$.

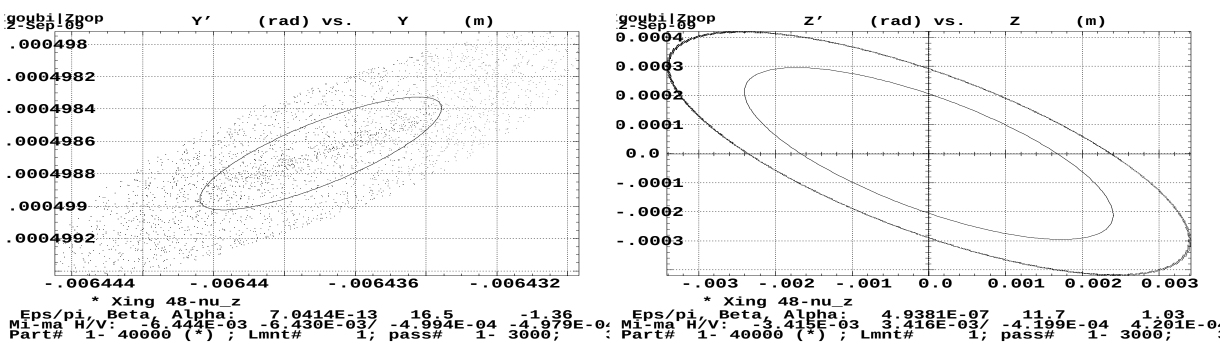



Figure 76: x-x' and z-z'.

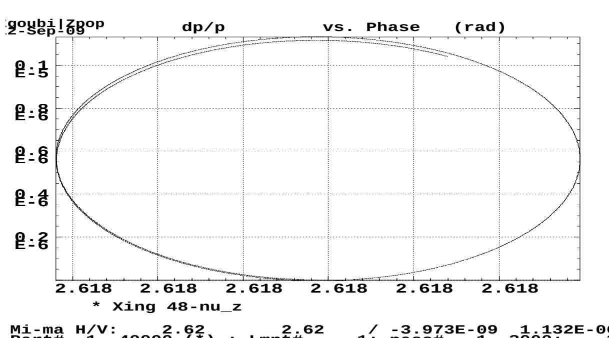



Figure 77: dp-phase.

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 2\,10^{-6}$

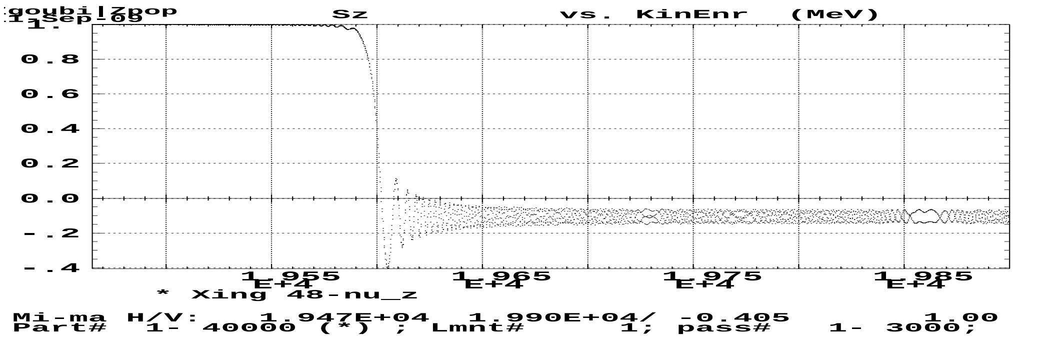



Figure 78: $S_z$ versus kinetic energy.

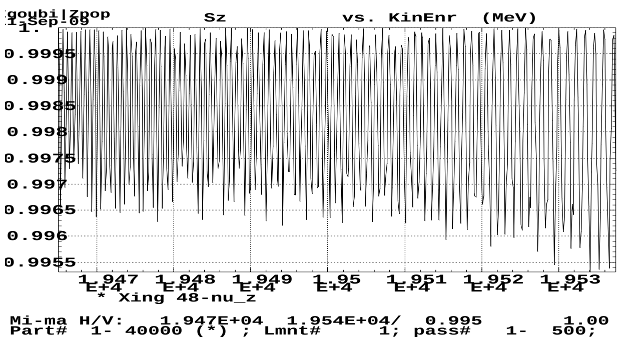



Figure 79: Zoom on initial $S_z$.

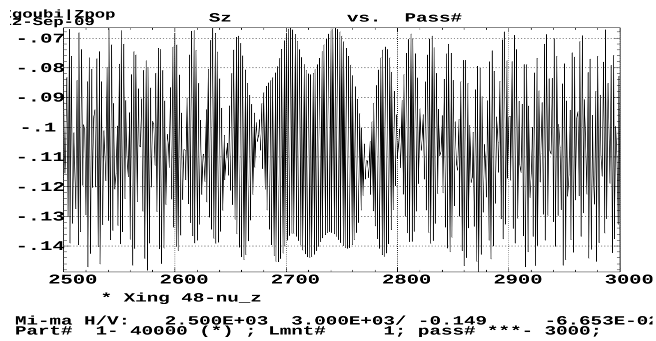



Figure 80: Zoom on final $S_z$.

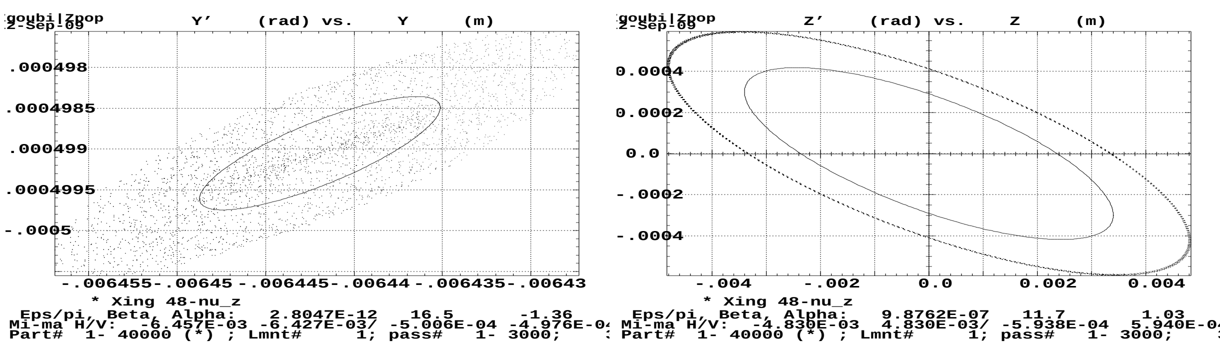



Figure 81: x-x' and z-z'.

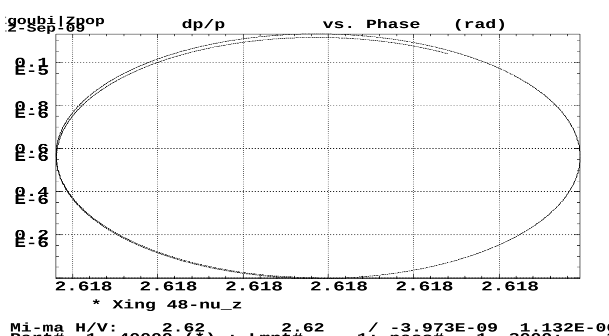



Figure 82: dp-phase.

### 4.1.8 $\gamma G = 48 - \nu_z$, bare lattice (all bends' $K_2$ coefficient off, all quadrupoles off)

Resonance strength computed from Zgoubi tracking may differ with results from the thin lens model in some cases of weak resonances, such as $\gamma G = 48 - \nu_z$. For that reason, in order to assess possible effect of sextupoles, we switch-off all sextupole components in all main bends. We also switch-off all tuning quadrupoles, the focusing is therefore due to the sole main bends' quadrupole component.

The following shows that the initial results are practically unchanged, the same resonance strength comes out of the ray-tracing.

Setting all sextupoles and quadrupoles off does not change much the ring parameters, 0-th and 1-st order machine parameters take the values given in Tab. 5. Earlier values are recalled for comparison.

A particle with vertical emittance $\epsilon_y/\pi = 10^{-6}$ m.rad is launched through the resonance. Spin motion upon crossing does not change much either, this can be observed from Fig. 83, by comparison with Fig. 58, page 42.

Table 5: Bare lattice parameters. Old ones (copied from Tab. 1) are recalled in the right column for comparison.

| | | New parameters | Old parameters |
|---|---|---|---|
| Reference momentum | (relative) | 1 | 1 |
| Circumference[4] | (m) | 807.0437 | 807.0438 |
| Qx, Qy | | 0.71066, 0.76412 | 0.71195, 0.76346 |
| *Periodic functions at "Begin AGS" :* | | | |
| $\beta_x,\ \beta_y$ | (m) | 19.7850, 11.7005 | 19.8432, 11.6751 |
| $\alpha_x,\ \alpha_y$ | | -1.585, -1.037 | -1.588, 1.033 |
| $D_x, D'_x$ | (m,-) | 2.043, 0.144 | 2.034, 0.144 |
| closed orbit, $x_{co}, x'_{co}$ | mm, mrad | -0.645, -0.50 | -6.43 -0.50[1,3] |
| $\gamma G = 48 - \nu_z$ | | 39.236 | 39.236 |
| kinetic energy on resonance | (GeV) | 19.6097 | 19.5958 |
| $B\rho$ on resonance | (T.m) | 68.4692 | 68.4229 |

## Tracking data

```
Zgoubi.dat, excerpts :

Xing 48-nu_z. Bare lattice, bends' sextu off, quadrupoles off.
'OBJET'
68.032e3
8
  1  1  1
 -6.453198E-03 -5.003742E-04     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   1e-6
  0  1  0.
 'SCALING'                                                                                                    3
 1  2
  MULTIPOL SBEN
  -1
  68.032
  1
  MULTIPOL QUAD
  -1
  0000000
  1
 'PARTICUL'                                                                                                   4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
....................
....................
 'MULTIPOL' SBEN      A1BF
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563e-20  0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|200|20    Dip  A1BF
            3   0.00000000000000        0.00000000000000      -1.175115045000000E-002
....................
....................
```

$\gamma G = 48 - \nu_z$ (19.6097 GeV) - $\epsilon_z/\pi = 10^{-6}$

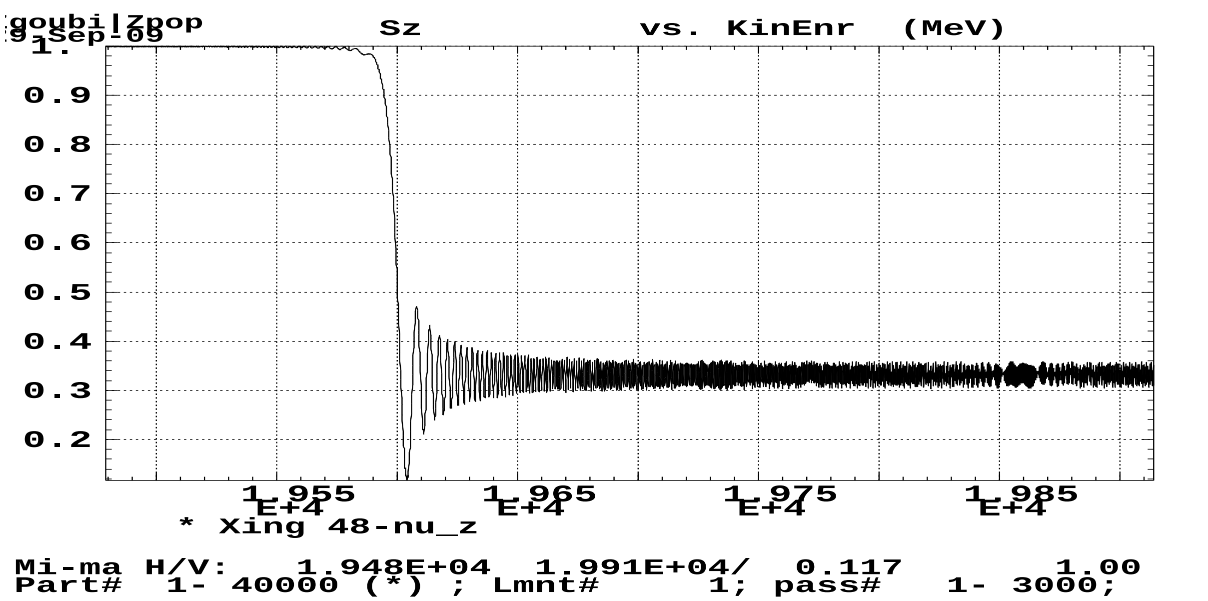



Figure 83: $S_z$ versus kinetic energy. See Fig. 58, page 42 for comparison.

### 4.1.9 $\gamma G = 36 + \nu_z$ (22.48832 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

Xing 36+nu
'OBJET'
77.64321e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   .2e-7 =epsilon_z/pi
  0  1  0.
 'SCALING'                                                                                                        3
 1  2
  MULTIPOL SBEN
  -1
  77.64321
  1
  MULTIPOL QUAD
  -1
  77.64321
  1
 'PARTICUL'                                                                                                       4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
.............
  'CAVITE'                                                                          85                           86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365       9cavitiesx32kV, phi_s=180-30deg
------------------
From zgoubi.res :

     Particle  properties :
                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285
              Reference  data :
                    rigidity (kG.cm)      :   77643.2
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   23276.8
                    energy, total (MeV)   :   23295.8
                    energy, kinetic (MeV) :   22357.5
                    beta = v/c            :  0.9991885714
                    gamma                 :   24.82835560
                    beta*gamma            :   24.80820916
```

## Strengths

From Figs. 20, 20 one gets

| $\epsilon_z/\pi$ $(10^{-6})$ | $A^2$ | $\lvert J_n\rvert^2$ $(10^{-5})$ | $A^2/\epsilon_z/\pi$ | $\lvert J_n\rvert^2/\epsilon_z/\pi$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|
| 0.002 | 0.1329602 | 0.37325392 | 6.648009E+07 | 1866 | 1. | 0.751 |
| 0.02 | 1.431838 | 4.0195435 | 7.1591912E+07 | 2009 | 1. | -0.522 |

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.002\,10^{-6}$

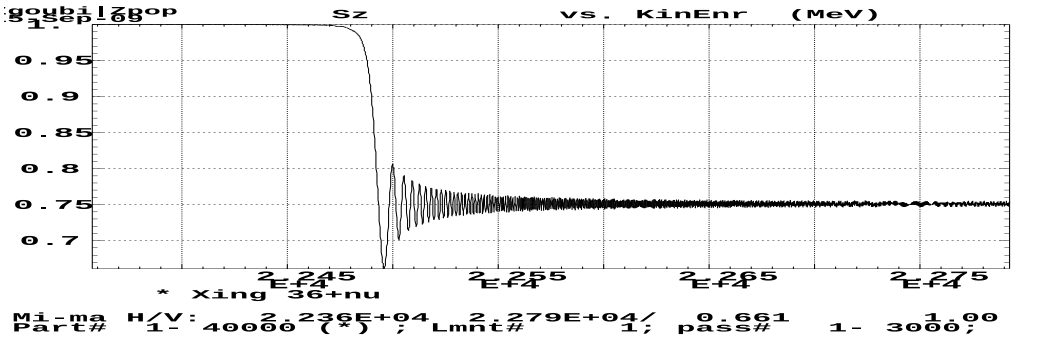



Figure 84: $S_z$ versus kinetic energy.

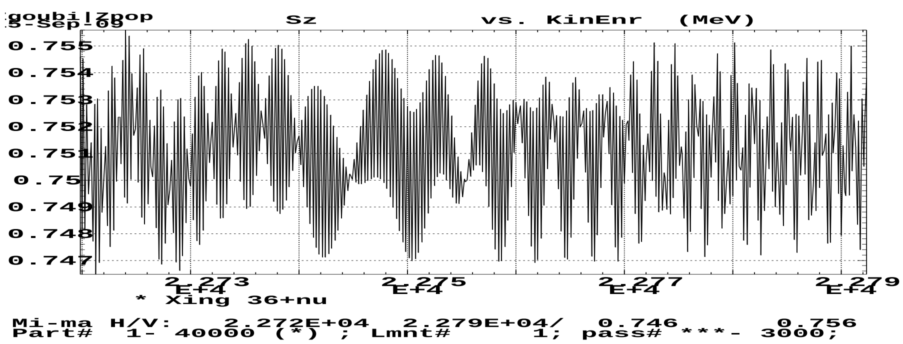



Figure 85: Zoom on final $S_z$.

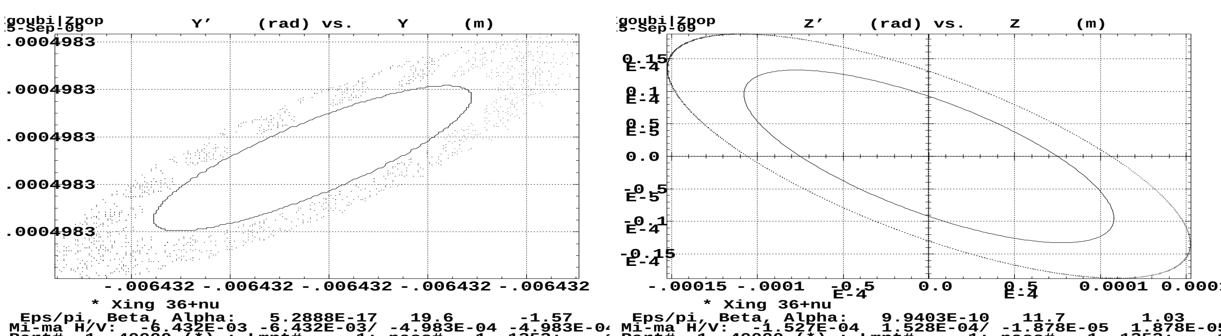



Figure 86: x-x' and z-z'.

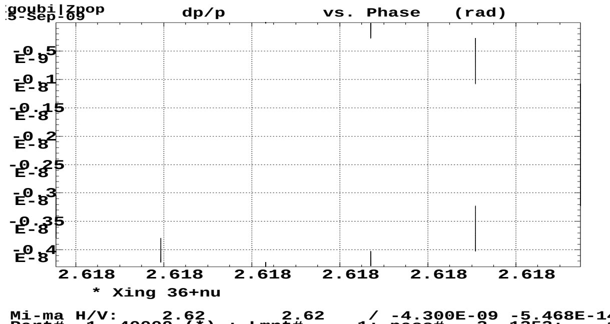



Figure 87: dp-phase.

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.02\,10^{-6}$

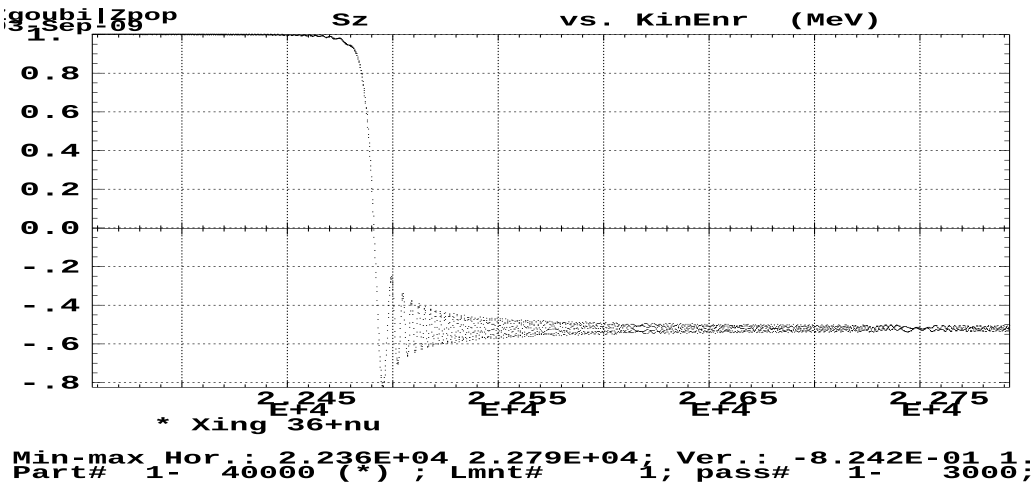



Figure 88: $S_z$ versus kinetic energy.

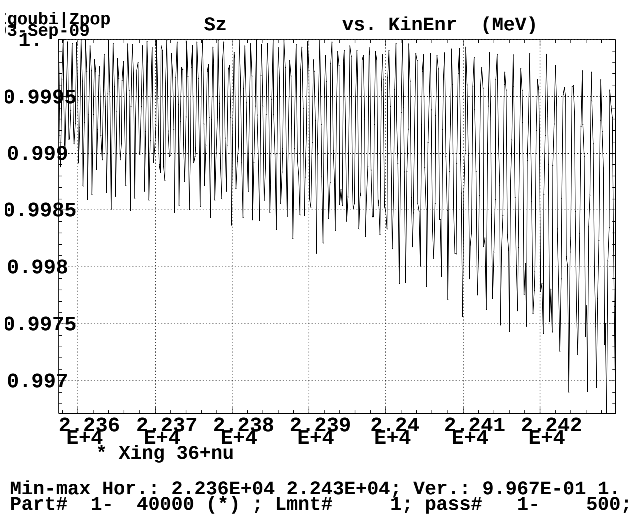



Figure 89: Zoom on initial $S_z$.

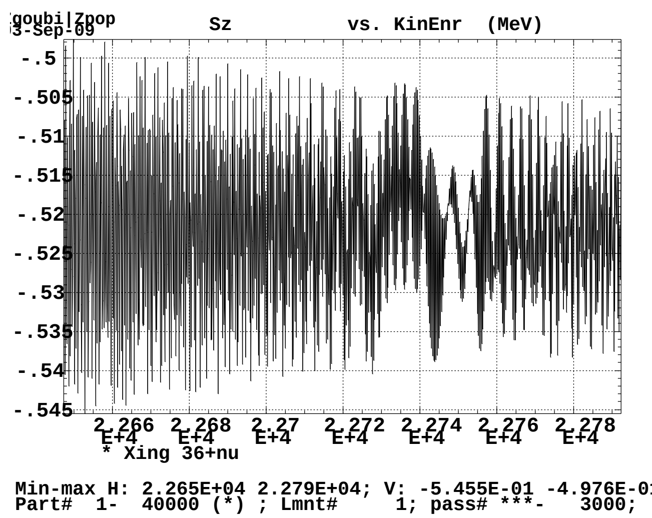



Figure 90: Zoom on final $S_z$.

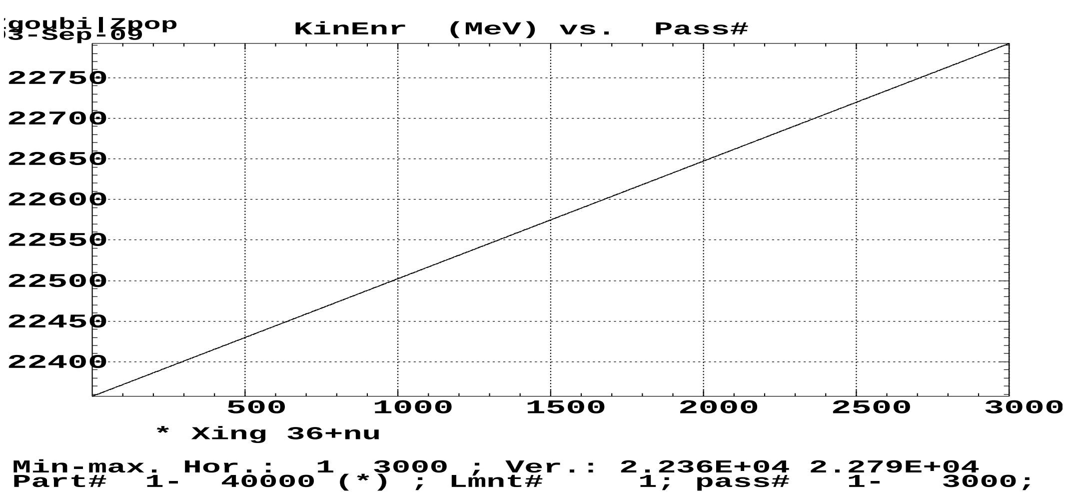



Figure 91: Kinetic E versus turn number.

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.02\,10^{-6}$

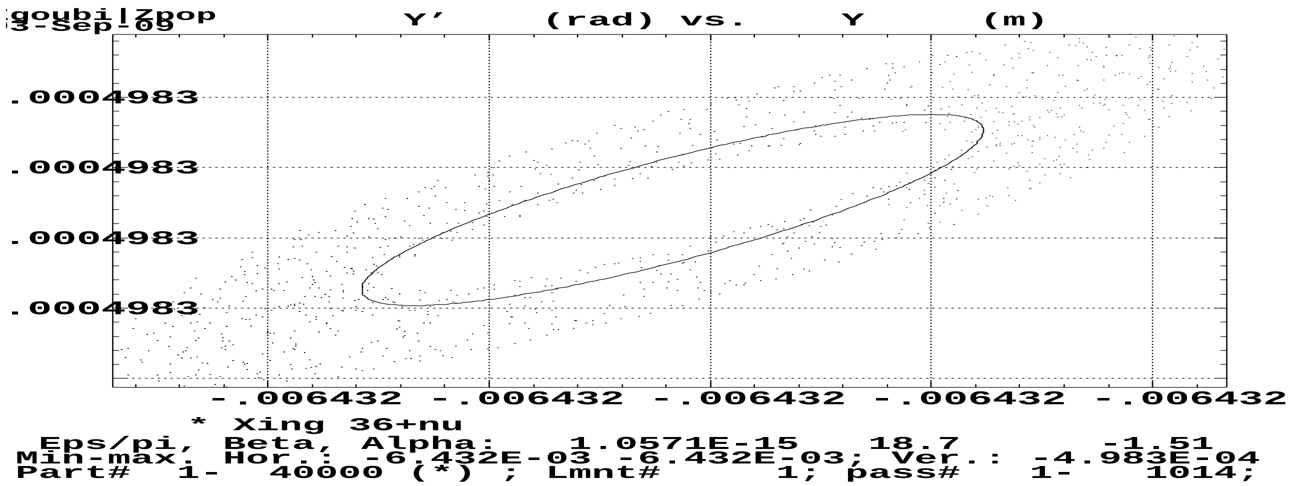



Figure 92: x-x'.

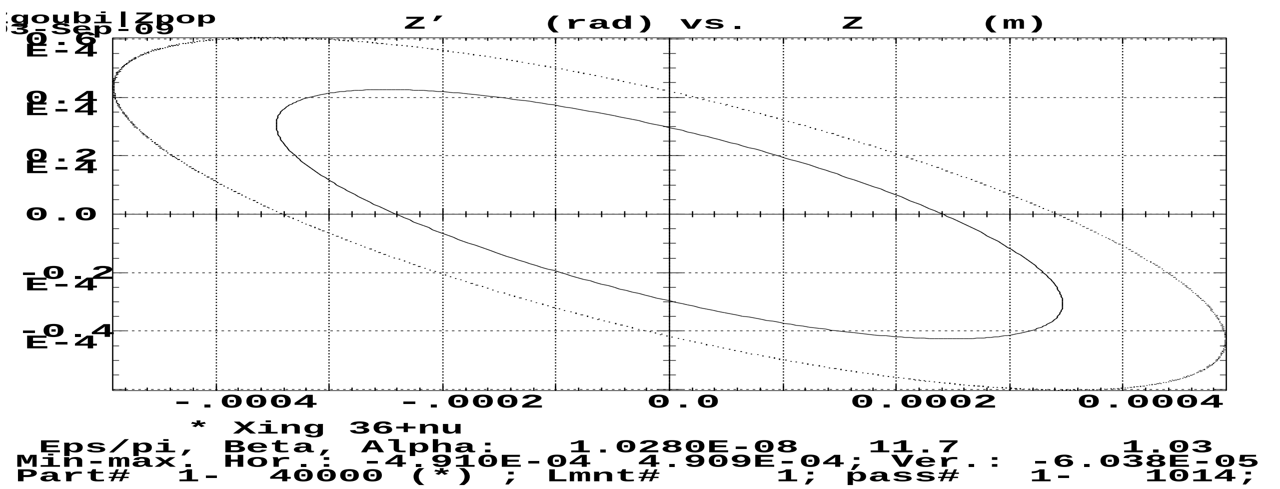



Figure 93: z-z'.

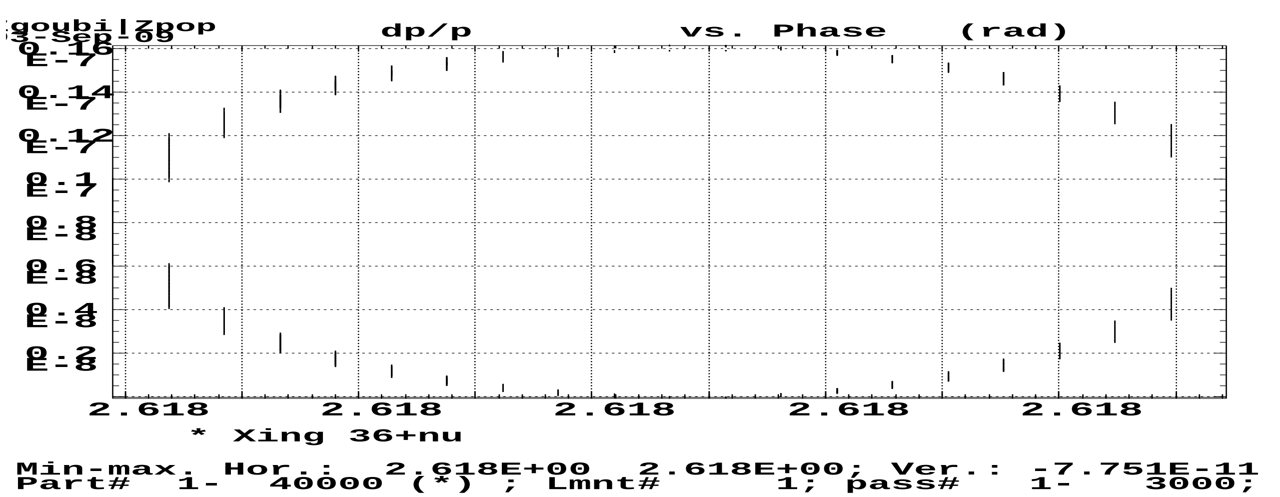



Figure 94: dp-phase.

## 4.2 Imperfection resonances

### 4.2.1 $\gamma G = 9$ (3.77180 GeV)

### Tracking data

```
Zgoubi.dat, excerpts :

Xing gammaG = 9
'OBJET'
14.95063e3
2
1  1
 -0.64333404 -0.49841318  7.55234854E-02  9.07085314E-02   0.0E+00  1.    'p'
1
'FAISCEAU'
 'SCALING'                                                                                                                   3
 1  3
  MULTIPOL VKIC
  -1
  14.95063                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  14.95063
  1
  MULTIPOL QUAD
  -1
  14.95063
  1
 'PARTICUL'                                                                                                                  4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
.............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  2.e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
.............
  'CAVITE'                                                              85                                       86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  0.5235987755982988731          9cavitiesx32kV, phi_s=30deg
   'MARKER'  #End
 'REBELOTE'
 1999  0.2 99
 'END'
------------------
From zgoubi.res :

     Particle  properties :
                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285
              Reference  data :
                    rigidity (kG.cm)      :   14121.6
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   4233.56
                    energy, total (MeV)   :   4336.29
                    energy, kinetic (MeV) :   3398.01
                    beta = v/c            :  0.9763099040
                    gamma                 :   4.621565007
                    beta*gamma            :   4.512079688
```

### Strengths

From Figs. 96, 97, Eq. 2, one gets

| kick (mrad) | $\hat{z}_{co}$ (mm) | $A^2$ | $\|J_n\|^2$ ($10^{-6}$) | $A^2/\hat{z}_{co}$ | $\|J_n\|^2/\hat{z}_{co}$ | $p_{init}$ | $p_{final}$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.093 | | | | 4.1295695 | | 0.997457 |
| 1 | 0.93 | | | | 4.1102805 | 1. | 0.762110 |
| | 2.77 | 1.224737 | 34.381555 | | | 0.992 | -0.412 |

$\gamma G = 9$ - $\hat{z}_{co} = 2.77$ mm

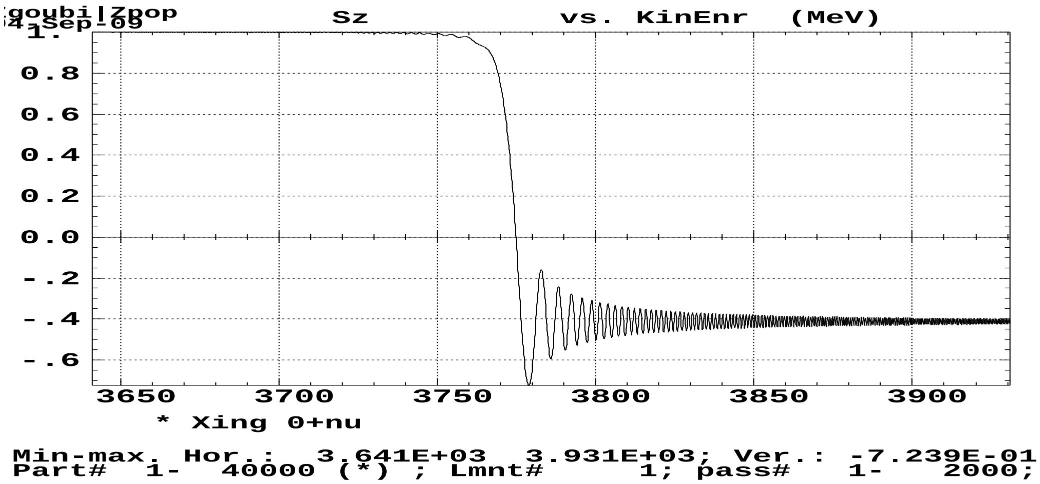



Figure 95: $S_z$ versus kinetic energy.

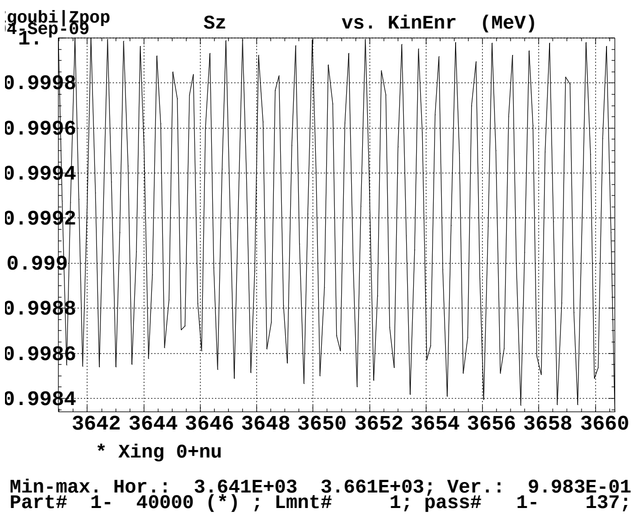



Figure 96: Zoom on initial $S_z$.

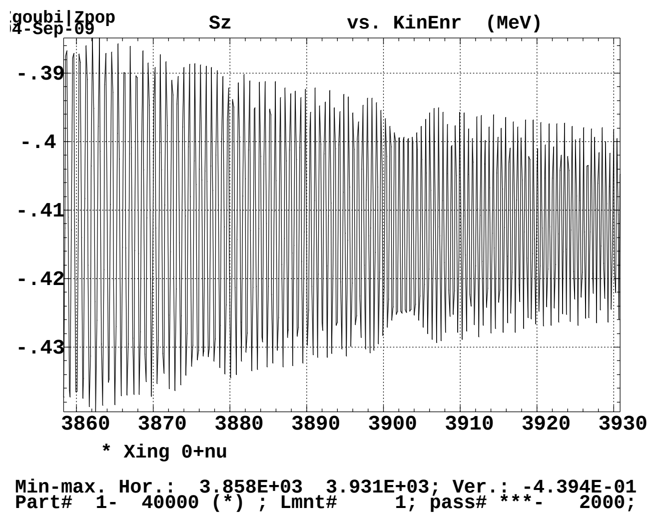



Figure 97: Zoom on final $S_z$.

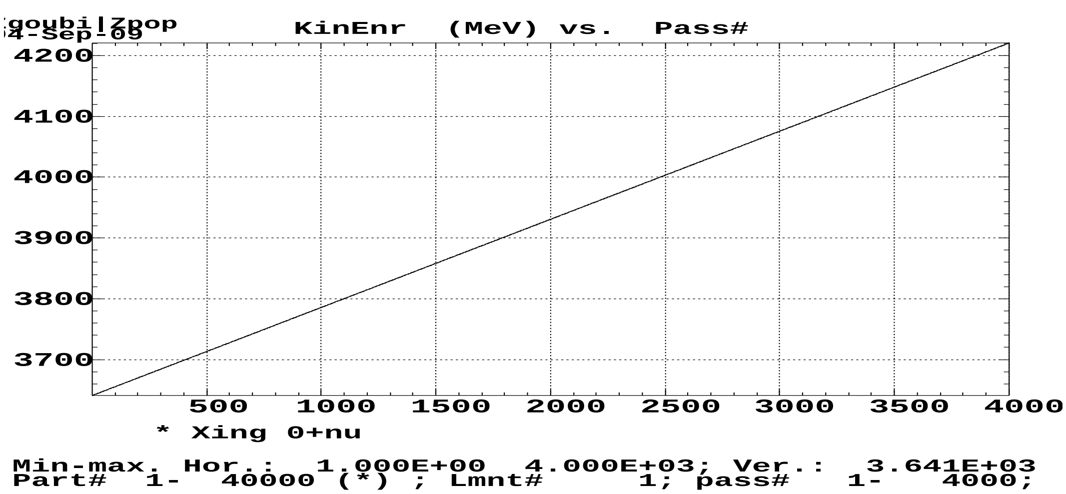



Figure 98: Kinetic E versus turn number.

$\gamma G = 9$ - $\hat{z}_{co} = 2.77$ mm

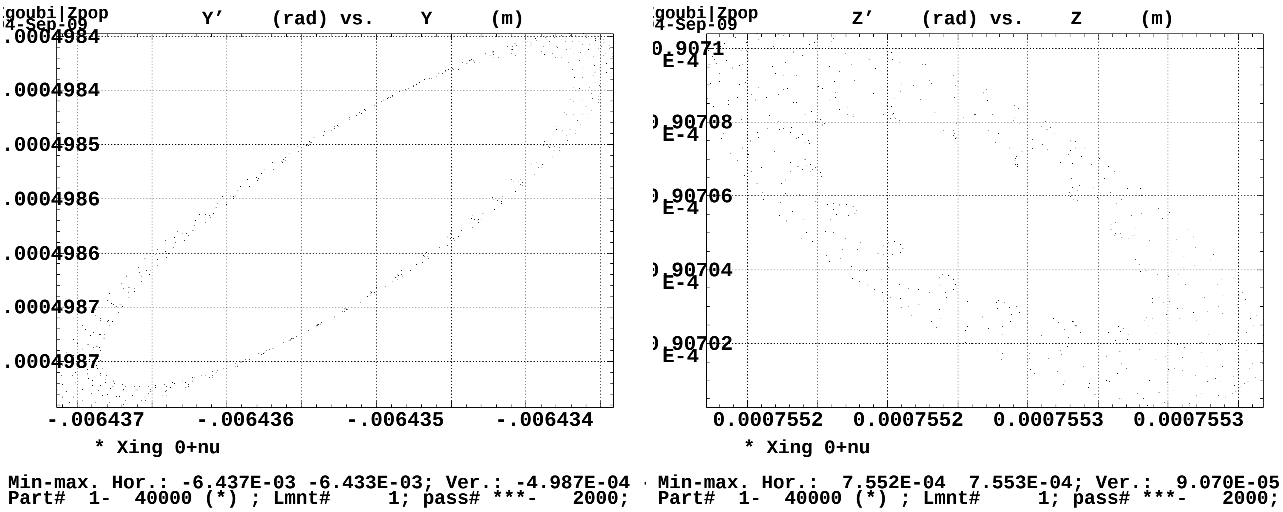



Figure 99: Left : x-x', right : z-z'.

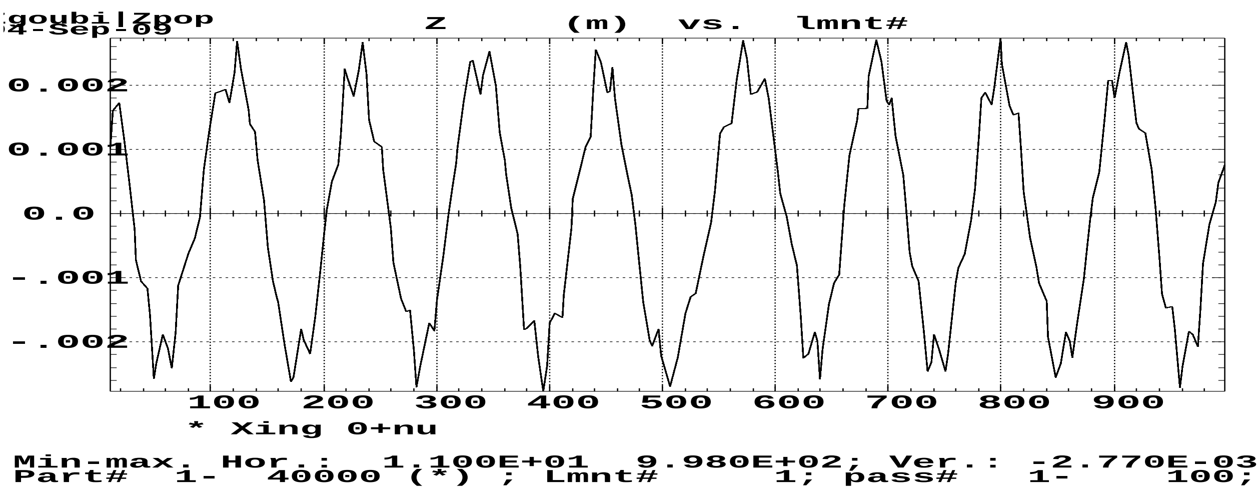



Figure 100: Vertical closed orbit along the ring circumference, versus pick-up number (about 990 P-Us over the 807 m circumference).

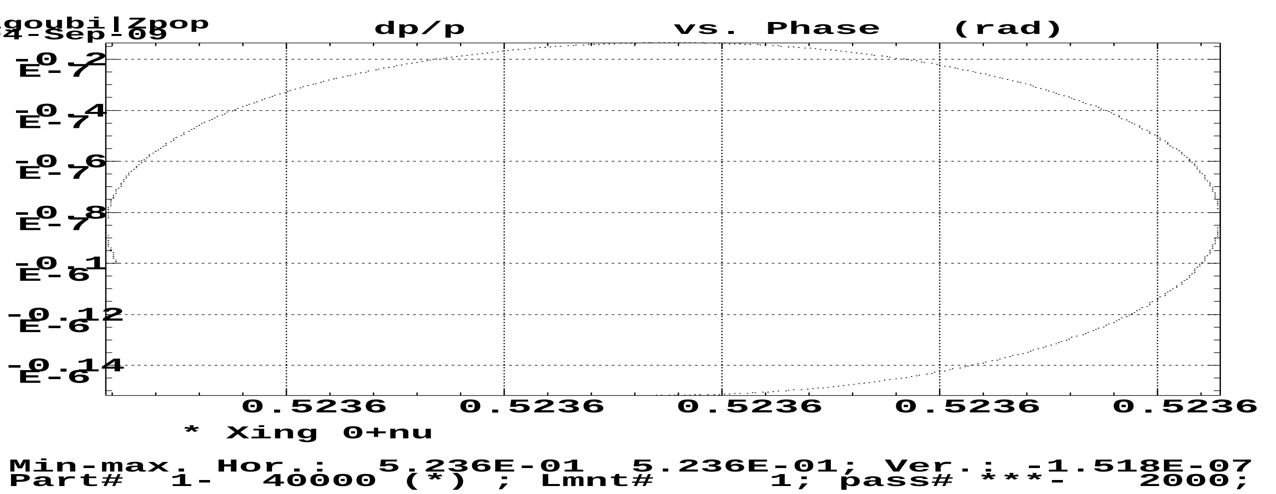



Figure 101: dp-phase.

### 4.2.2 $\gamma G = 12$ (5.341830 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

Xing gammaG = 12
'OBJET'
20.27157e3
2
1  1
 -0.65068584 -0.50477891    0.37676032  0.45393654  0.0E+00  1.    'p'
1
 'SCALING'                                                                                                  3
 1  3
  MULTIPOL VKIC
  -1
  20.27157                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  20.27157
  1
  MULTIPOL QUAD
  -1
  20.27157
  1
 ............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  10.e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
.............
  'CAVITE'                                                    85                                        86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  0.5235987755982988731          9cavitiesx32kV, phi_s=30deg
------------------
From zgoubi.res :

     Particle  properties :
                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285
              Reference  data :
                    rigidity (kG.cm)      :   20271.6
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   6077.26
                    energy, total (MeV)   :   6149.27
                    energy, kinetic (MeV) :   5211.00
                    beta = v/c            :  0.9882907009
                    gamma                 :   6.553821507
                    beta*gamma            :   6.477080850

               Accelerating  cavity
                OPTION 2
                    Orbit  length         =      807.0     m
                    RF  harmonic          =      12.00
                    Peak  voltage         =     2.9000E+05 V
                    RF  frequency         =     4.4054E+06 Hz
                    Synchronous  phase    =     0.5236     rd
                    Isochronous  time     =     2.7239E-06 s
                    qV.SIN(Phi_s)         =     0.1450     MeV
                    cos(Phi_s)            =     0.8660
                    Nu_s/sqrt(alpha)      =     8.8318E-03
                    dp-acc*sqrt(alpha)    =     9.2558E-04
          Spin stuff :
                    alpha = G*dgamma/dtta =     4.4096356E-05
                    B-dot *rho(m)         =      179.7     T/s
```

## Strength

From Figs. 103, 104 one gets

$p_{init} \approx 0.9999, \quad p_{final} \approx 5.674E-2$

Eq. 2 yields

$A^2 = 0.6379532$

$|J_n|^2 = 9.9891440E-06$

$\gamma G = 12$ (5.341830 GeV)

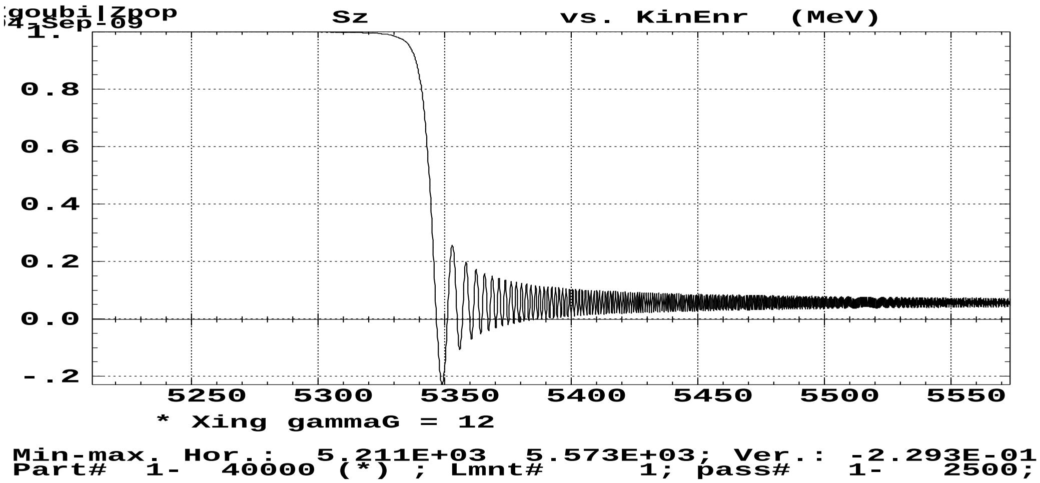



Figure 102: $S_z$ versus kinetic energy.

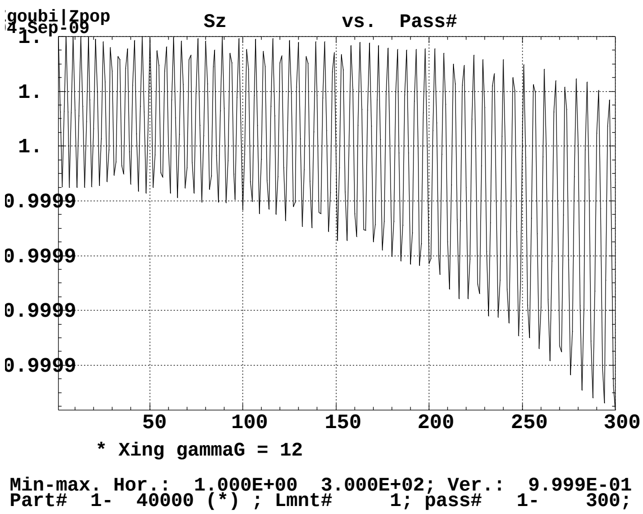



Figure 103: Zoom on initial $S_z$.

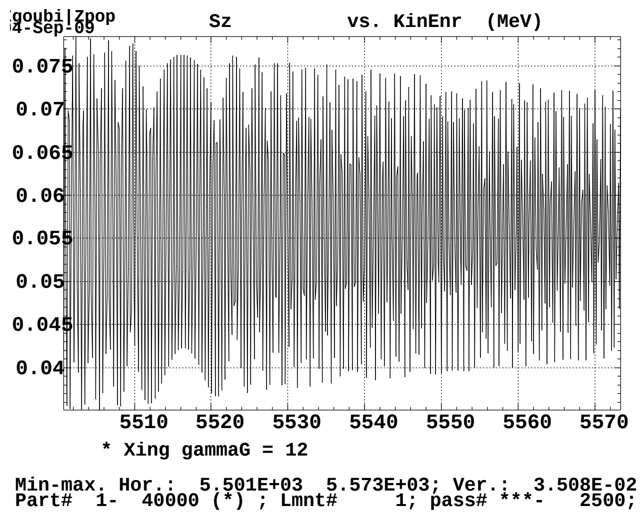



Figure 104: Zoom on final $S_z$.

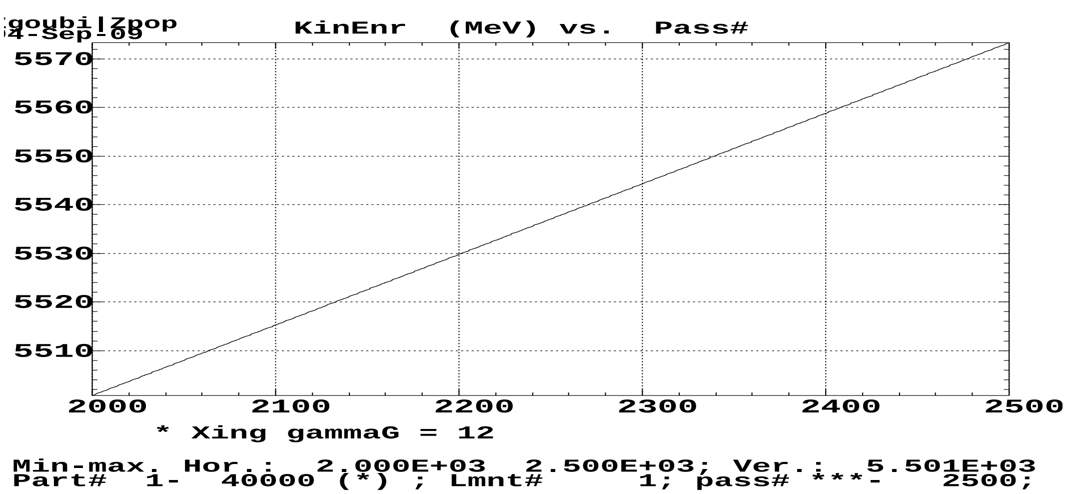



Figure 105: Kinetic E versus turn number.

$\gamma G = 12$ (5.341830 GeV)

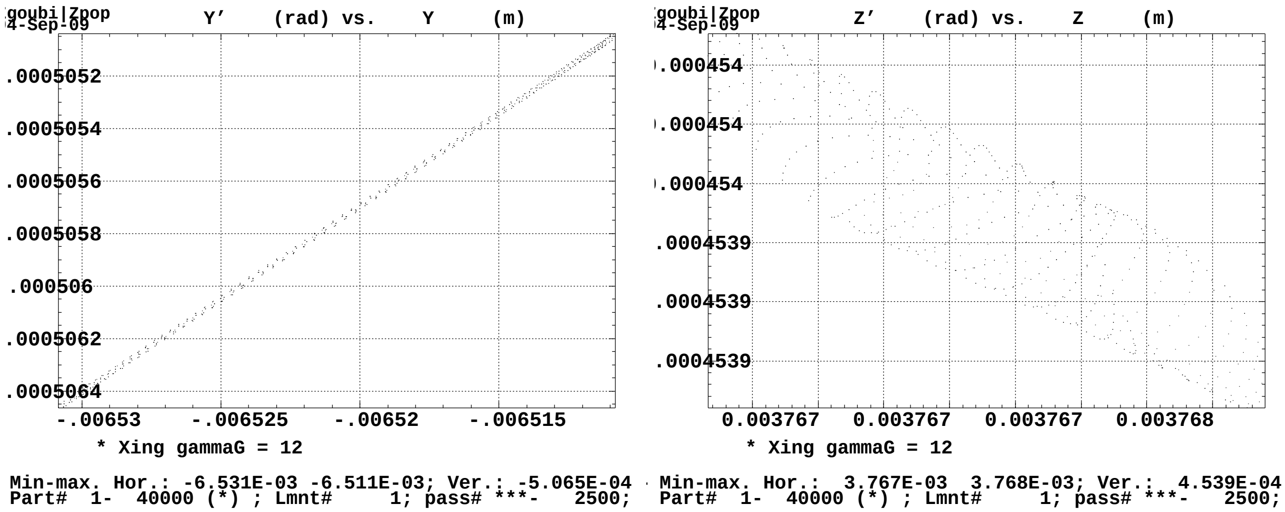



Figure 106: Left : x-x', right : z-z'.

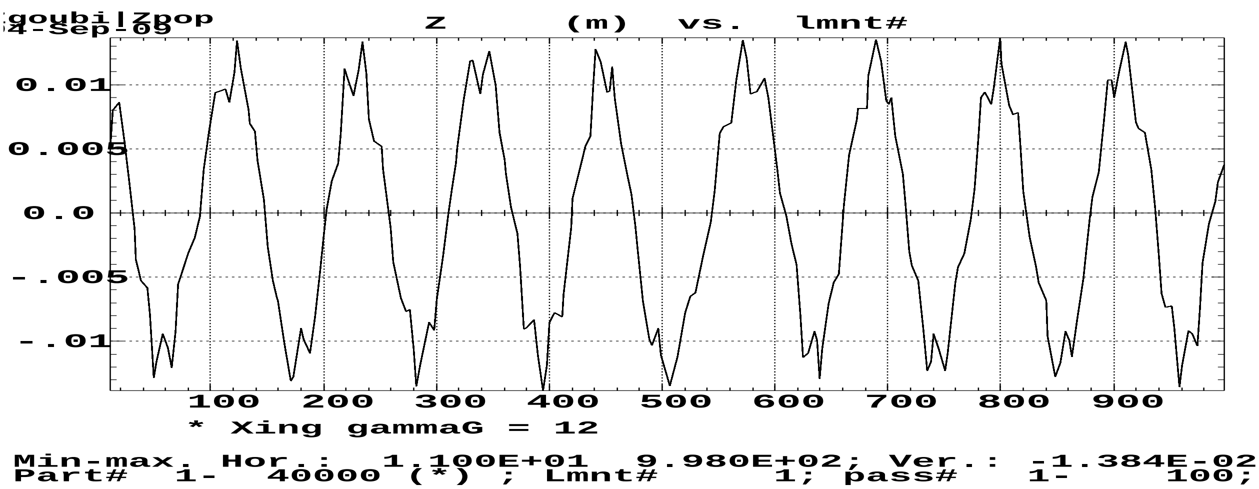



Figure 107: z closed orbit along the ring circumference versus pick-up number (about 990 P-Us over the 807 m circumference).

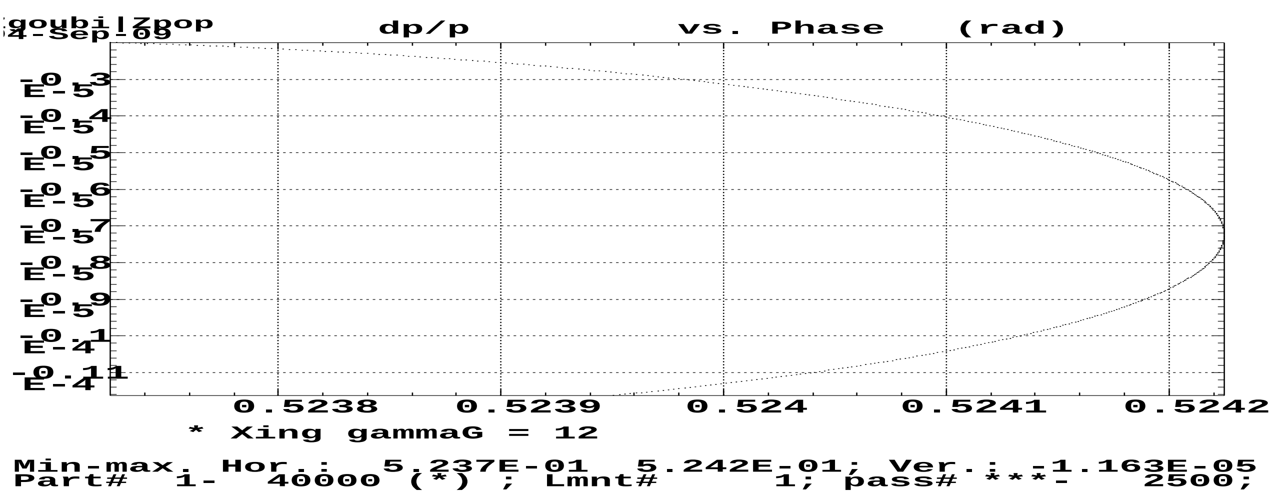



Figure 108: dp-phase.

### 4.2.3 $\gamma G = 13$ (5.865171 GeV)

#### Tracking data

```
Zgoubi.dat, excerpts :

Xing gammaG = 13
'OBJET'
22.03628e3
2
1  1
 -0.65068584 -0.50477891    0.37676032  0.45393654  0.0E+00  1.    'p'
1
'FAISCEAU'
 'SCALING'                                                                                                                    3
 1  3
  MULTIPOL VKIC
  -1
  22.036287                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  22.03628
  1
  MULTIPOL QUAD
  -1
  22.03628
  1
 'PARTICUL'                                                                                                                   4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
 ............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  10.e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 ............
  'CAVITE'                                                              85                                      86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  0.5235987755982988731          9cavitiesx32kV, phi_s=30deg
------------------
From zgoubi.res :

     Particle  properties :

                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285

              Reference  data :
                    rigidity (kG.cm)      :   22036.3
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   6606.31
                    energy, total (MeV)   :   6672.61
                    energy, kinetic (MeV) :   5734.34
                    beta = v/c            :  0.9900642821
                    gamma                 :   7.111591821
                    beta*gamma            :   7.040933050
```

#### Strength

From Figs. 110, 111 one gets

$p_{init} \approx 1.,\ \ p_{final} \approx 0.1574500$

Eq. 2 yields

$A^2 = 0.5469143$

$|J_n|^2 = 1.5353313E - 05$

$\gamma G = 13$ **(5.865171 GeV)**

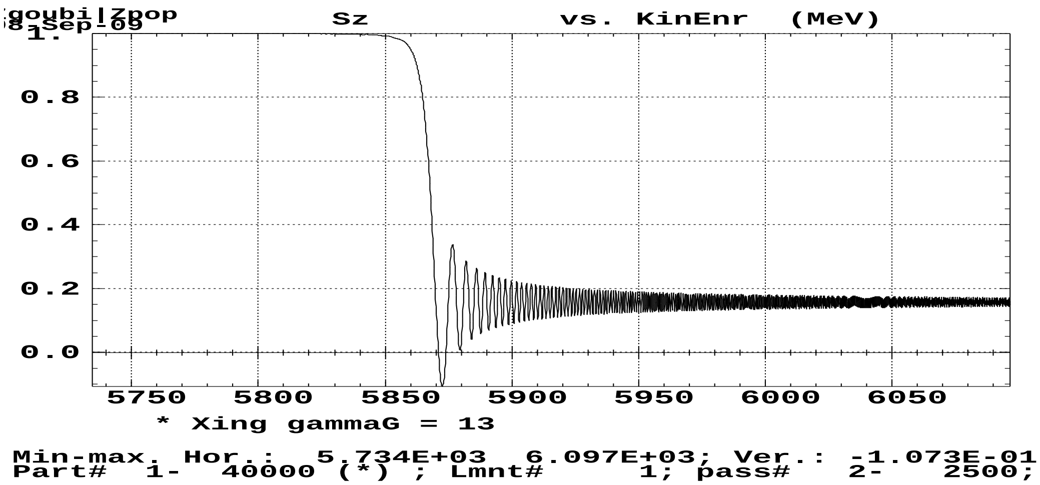



Figure 109: $S_z$ versus kinetic energy.

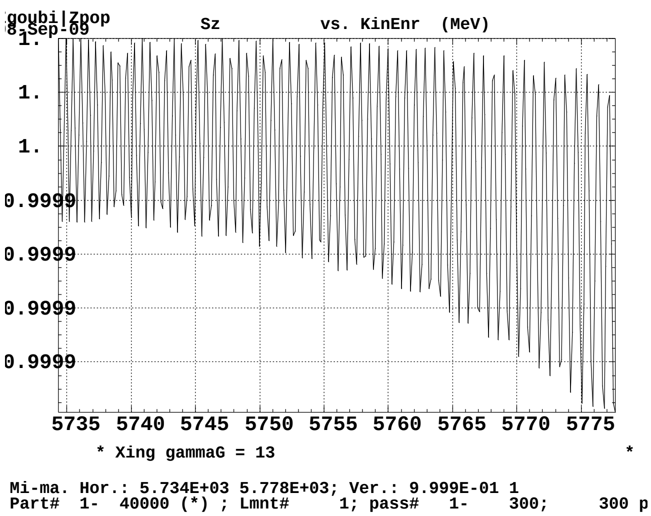



Figure 110: Zoom on initial $S_z$.

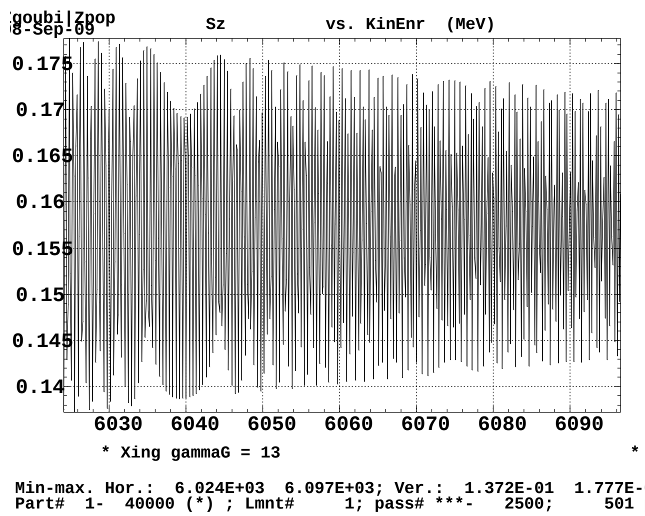



Figure 111: Zoom on final $S_z$.

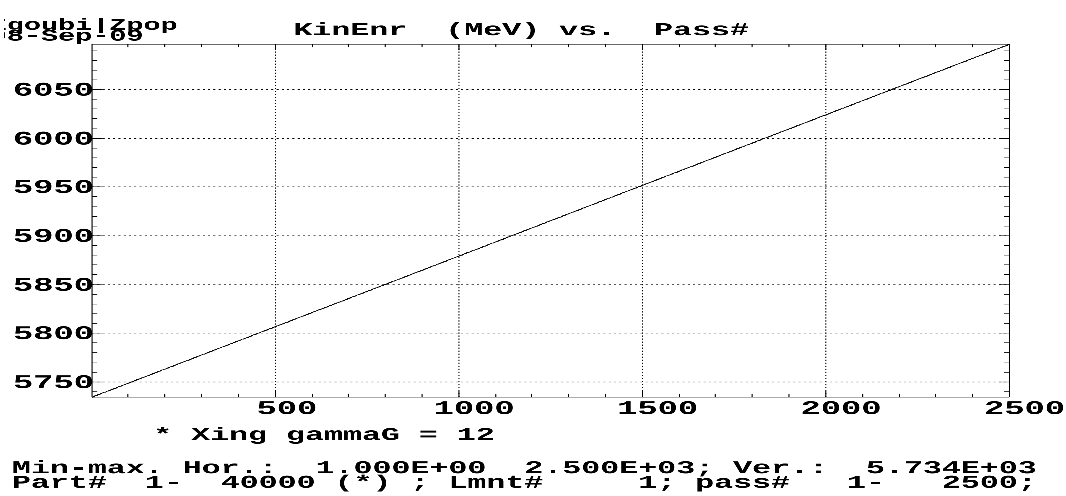



Figure 112: Kinetic E versus turn number.

$\gamma G = 13$ **(5.865171 GeV)**

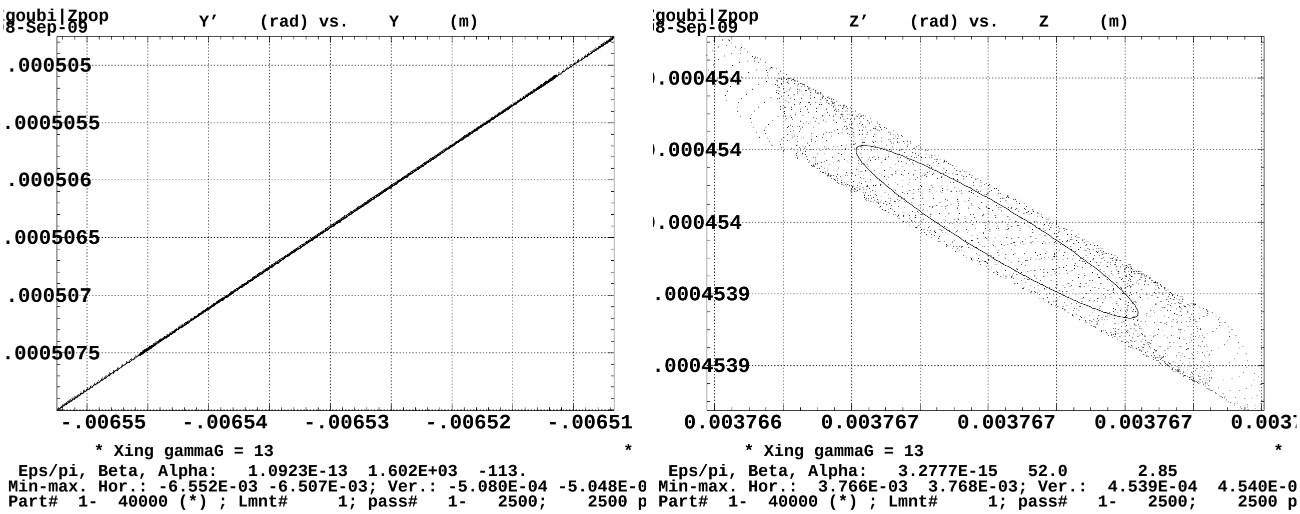



Figure 113: Left : x-x', right : z-z'.

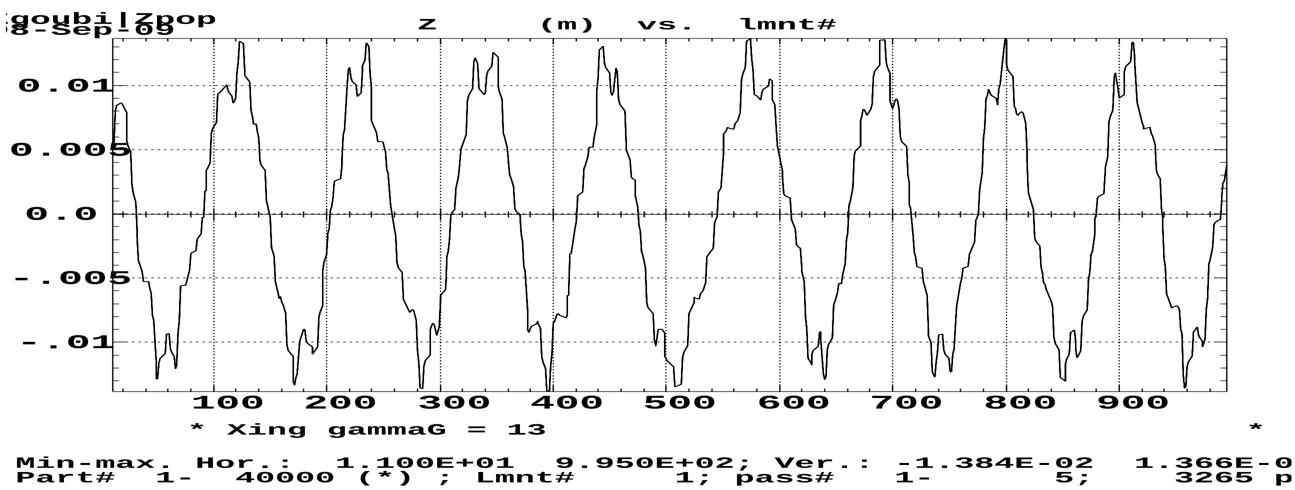



Figure 114: z closed orbit along the ring circumference (horizontal axis is pick-up number).

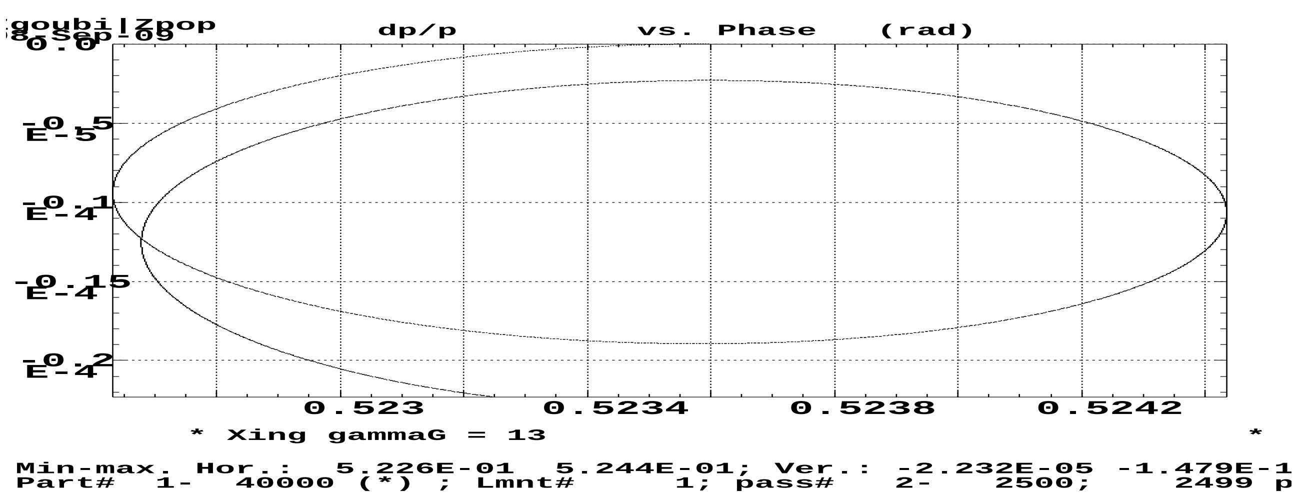



Figure 115: dp-phase.

### 4.2.4 $\gamma G = 23$ (11.09859 GeV)

### Tracking data

```
Zgoubi.dat, excerpts :

Xing gammaG = 23
'OBJET'
39.59072e3
2
1  1
 -0.65068584 -0.50477891    0.37676032  0.45393654  0.0E+00  1.    'p'
1
'FAISCEAU'
 'SCALING'                                                                                                                  3
 1  3
  MULTIPOL VKIC
  -1
  39.59072                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  39.59072
  1
  MULTIPOL QUAD
  -1
  39.59072
  1
 'PARTICUL'                                                                                                                 4
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
 .............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  10.e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 .............
 'CAVITE'                                                                        85                                        86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365          9cavitiesx32kV, phi_s=180-30deg
```

### Strength

From Figs. 117, 118 one gets

$p_{init} \approx 0.9975, \quad p_{final} \approx -0.647$

Eq. 2 yields

$A^2 = 1.734326$

$|J_n|^2 = 4.8687059E - 05$

$\gamma G = 23$ **(11.09859 GeV)**

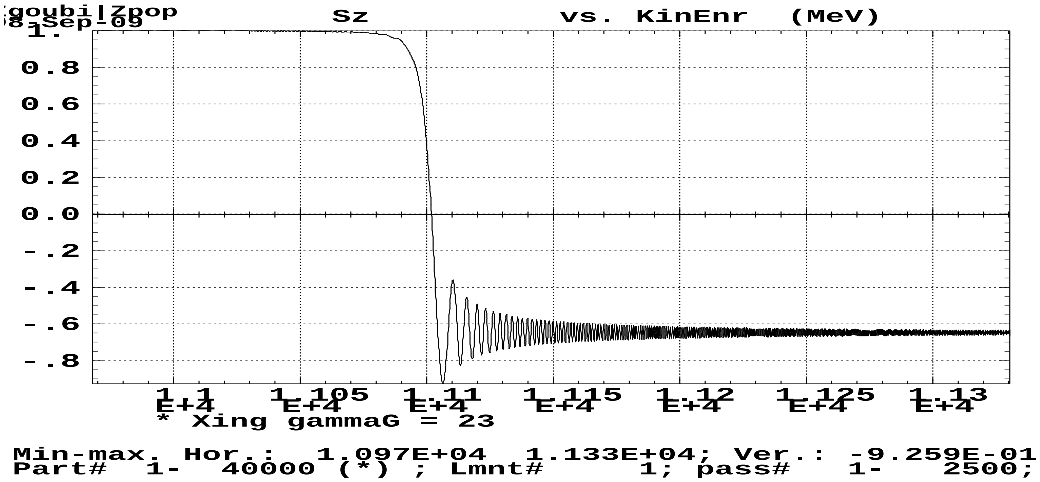



Figure 116: $S_z$ versus kinetic energy.

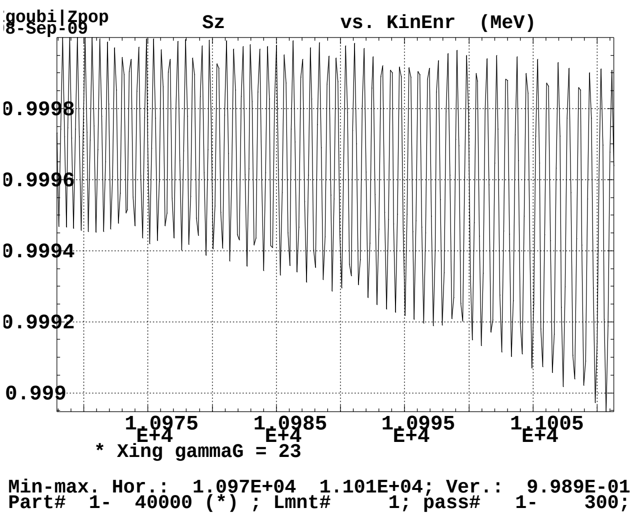



Figure 117: Zoom on initial $S_z$.

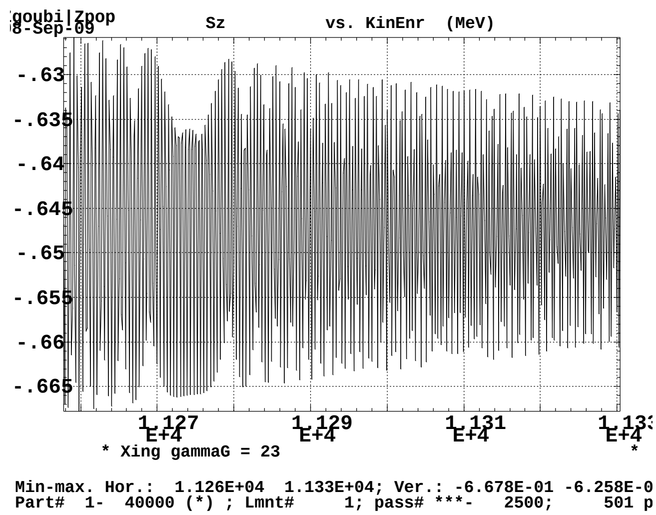



Figure 118: Zoom on final $S_z$.

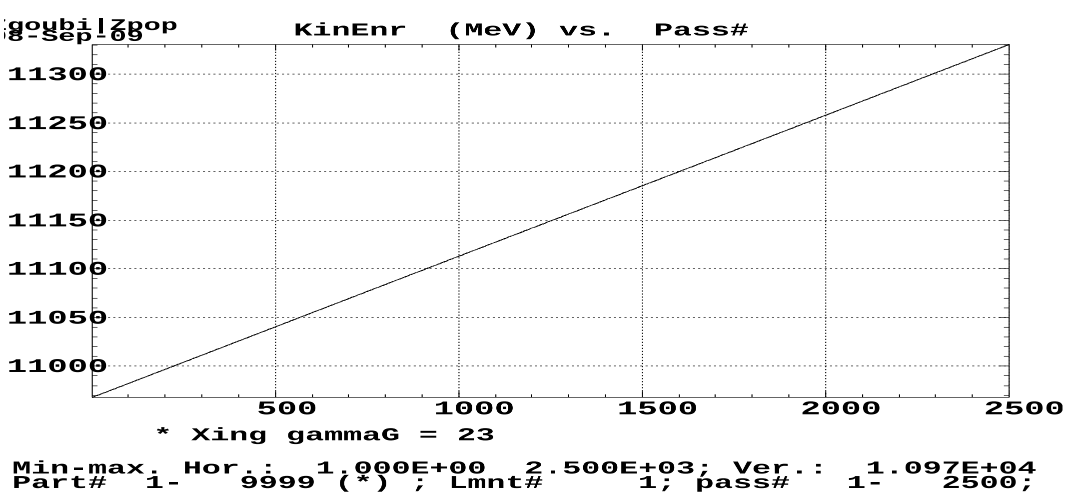



Figure 119: Kinetic E versus turn number.

$\gamma G = 23$ **(11.09859 GeV)**

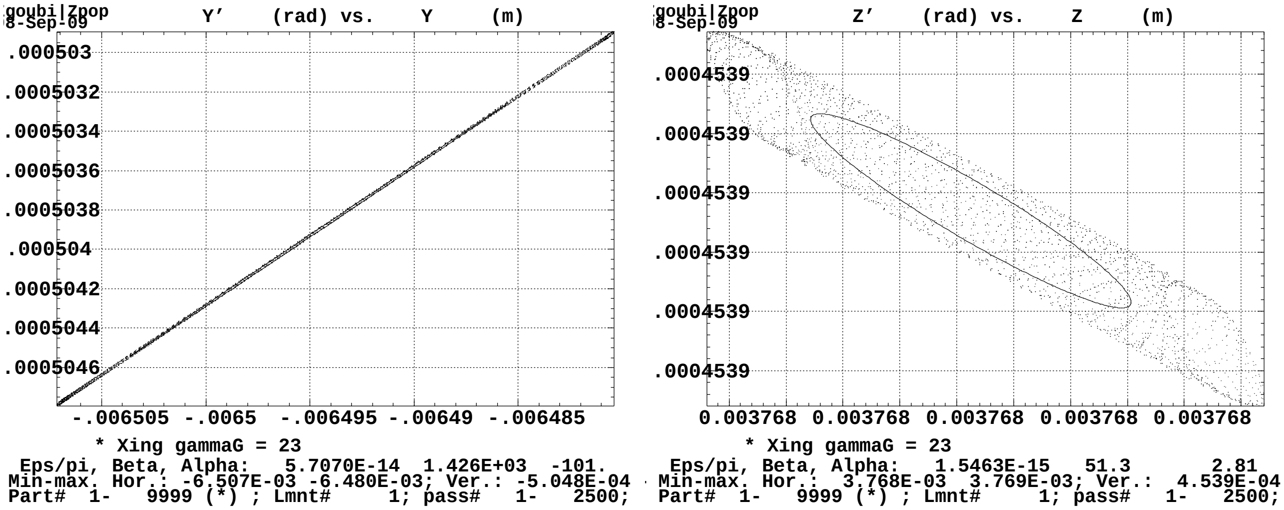



Figure 120: Left : x-x', right : z-z'.

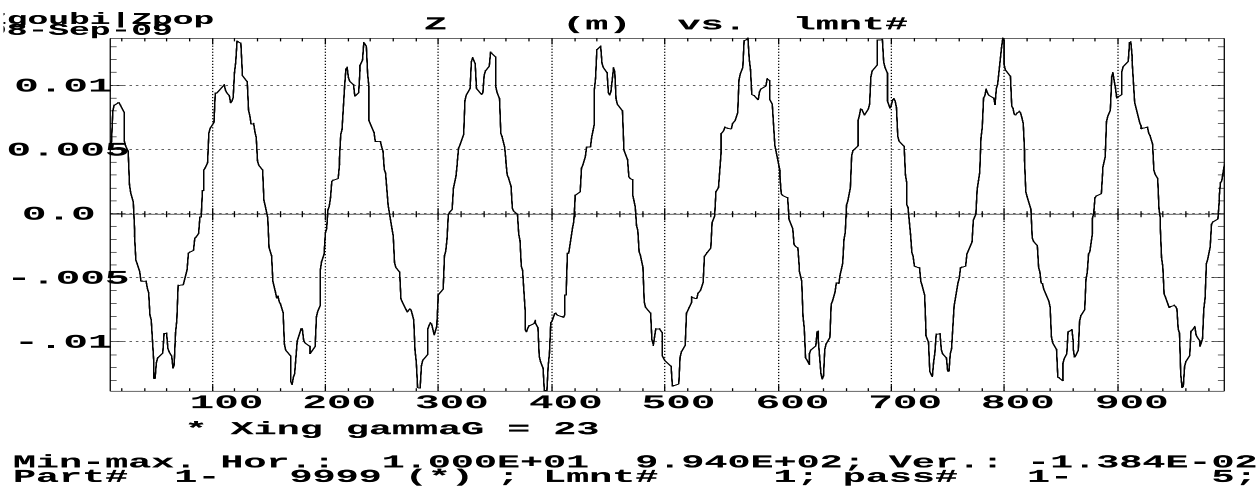



Figure 121: z closed orbit along the ring circumference (horizontal axis is pick-up number).

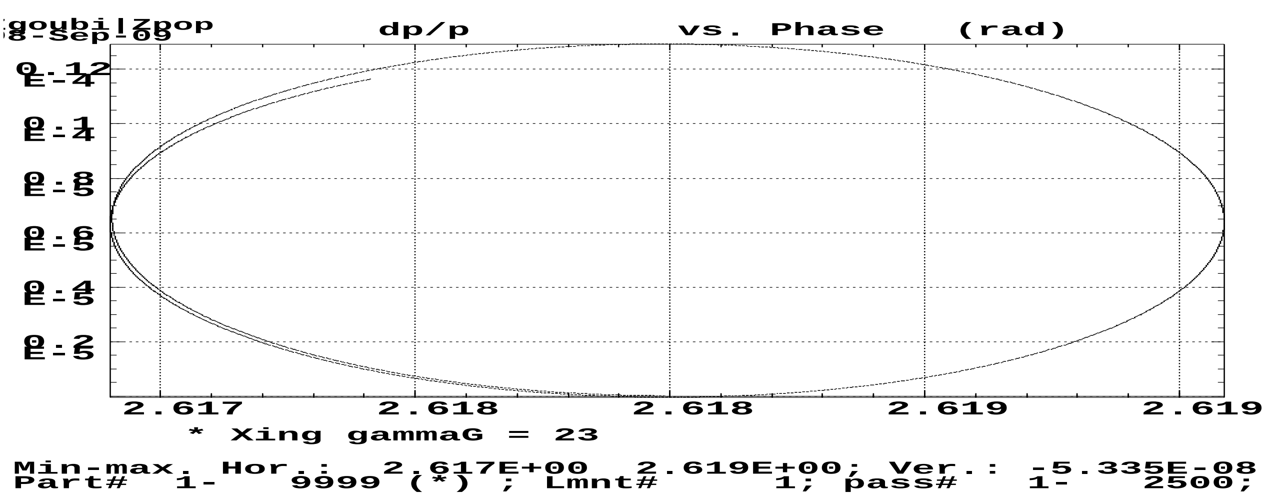



Figure 122: dp-phase.

### 4.2.5 $\gamma G = 27$ (13.19196 GeV)

## Tracking data

```
Zgoubi.dat, excerpts :

'OBJET'
46.59195e3
2
1  1
 -0.65068584 -0.50477891    0.037676032  0.045393654  0.0E+00  1.    'p'
1
'FAISCEAU'
 'SCALING'                                                                                  3
 1  3
  MULTIPOL VKIC
  -1
  46.59195                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  46.59195
  1
  MULTIPOL QUAD
  -1
  46.59195
  1
 ............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  1.e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 ............
 'CAVITE'                                                          85                                        86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365          9cavitiesx32kV, phi_s=180-30deg
```

---

```
from zgoubi.res :

     Particle  properties :

                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285
              Reference  data :
                    rigidity (kG.cm)      :   46591.9
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   13967.9
                    energy, total (MeV)   :   13999.4
                    energy, kinetic (MeV) :   13061.1
                    beta = v/c            :  0.9977514753
                    gamma                 :   14.92039915
                    beta*gamma            :   14.88685026
```

## Strength

From Figs. 124, 125 one gets

$p_{init} \approx 0.9998, \quad p_{final} \approx -0.167355$

Eq. 2 yields

$A^2 = 0.8763353$

$|J_n|^2 = 2.4601020E - 05$

$\gamma G = 27$ **(13.19196 GeV)**

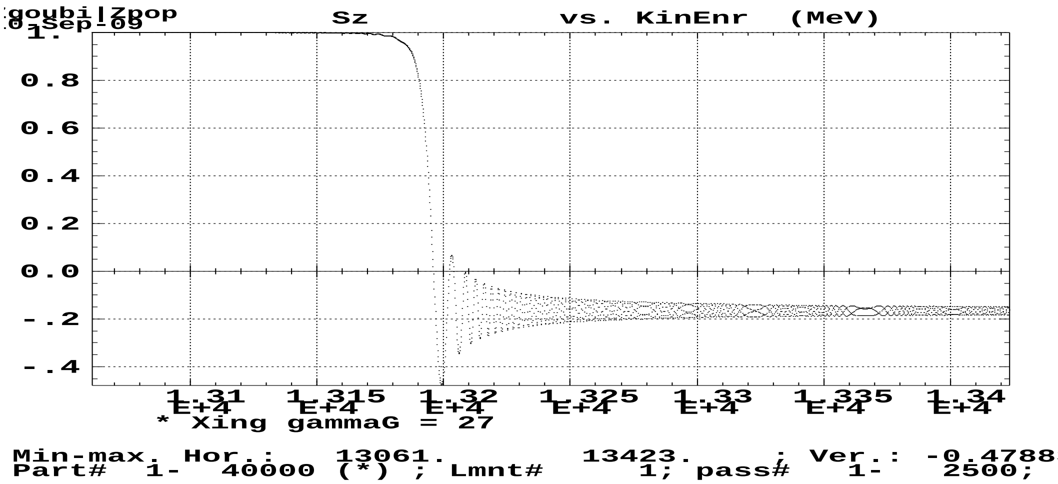



Figure 123: $S_z$ versus kinetic energy. Check : 36-Qy next to gG=27 is on the path at E=13.31575 GeV, yet not visible as expected since $\epsilon_z \approx 0$.

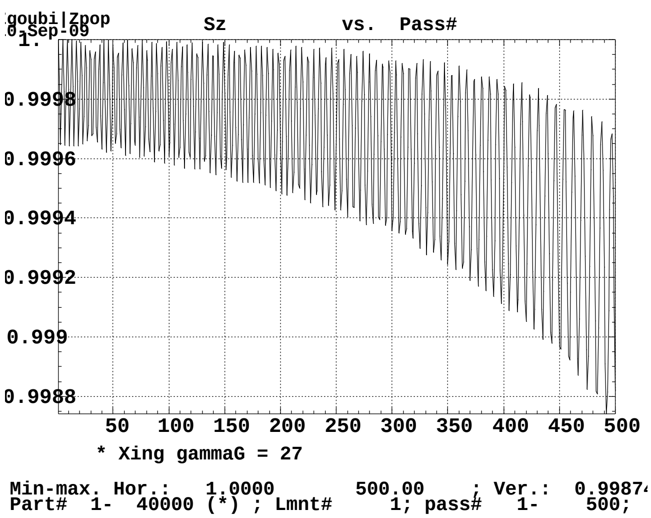



Figure 124: Zoom on initial $S_z$.

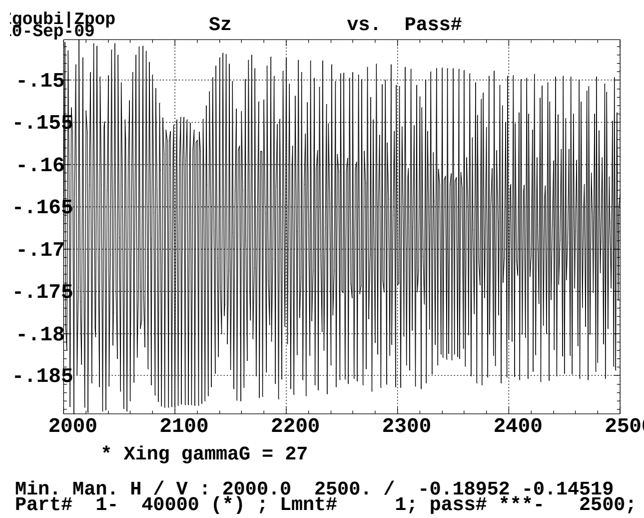



Figure 125: Zoom on final $S_z$.

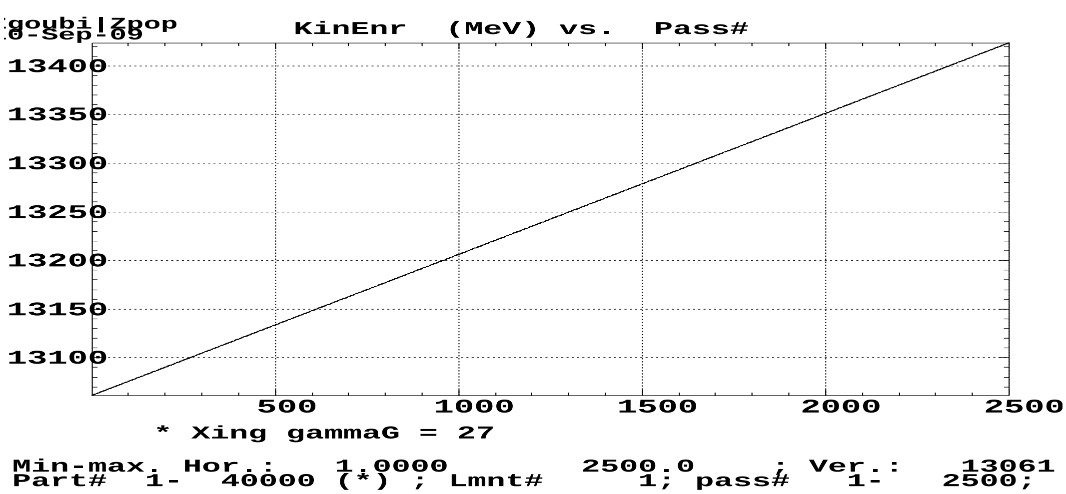



Figure 126: Kinetic E versus turn number.

$\gamma G = 27$ **(13.19196 GeV)**

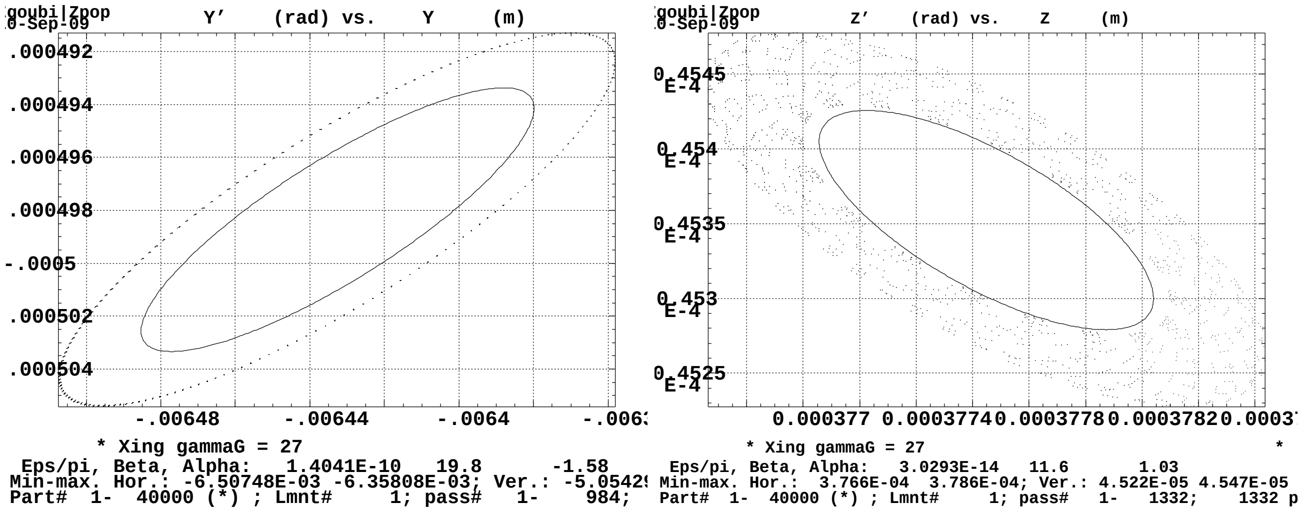



Figure 127: Left : x-x', right : z-z'.

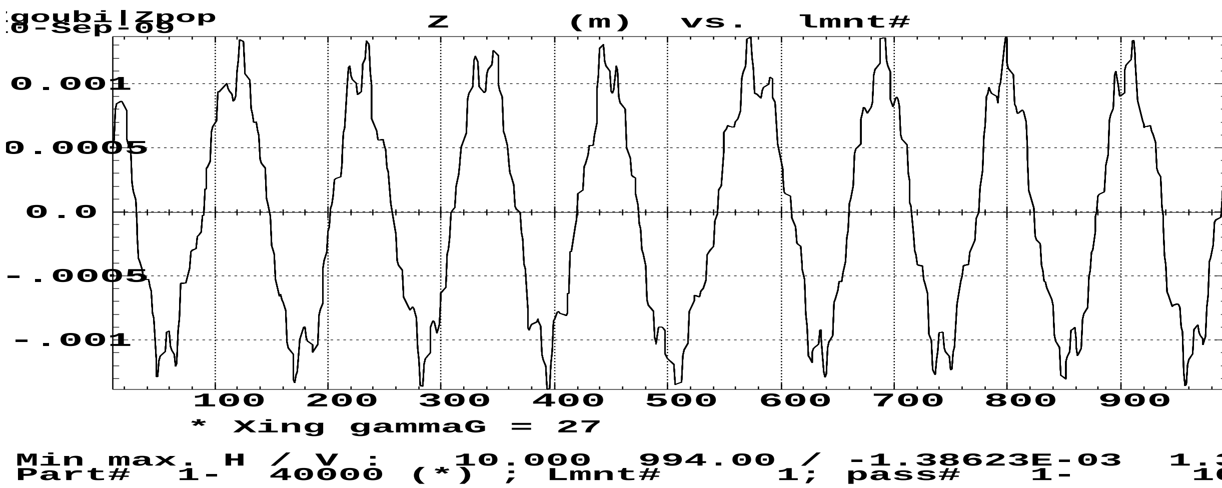



Figure 128: z closed orbit along the ring circumference (horizontal axis is pick-up number), 9 turns over the ring are superimposed.

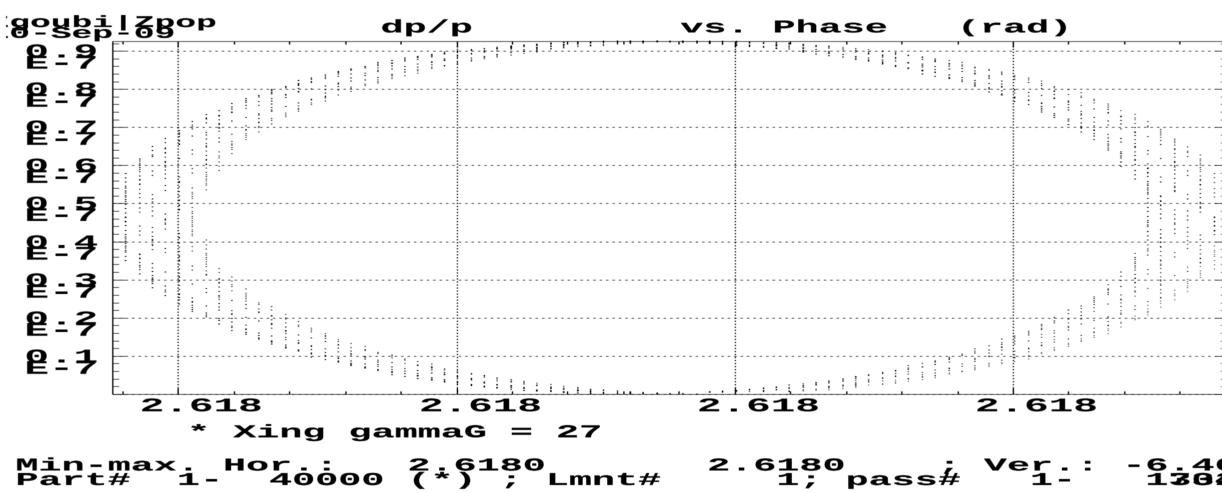



Figure 129: dp-phase.

### 4.2.6 $\gamma G = 45$ (22.61211 GeV)

#### Tracking data

```
Zgoubi.dat, excerpts :

Xing gammaG = 45
'OBJET'
78.05648e3
2
1  1
 -0.65068584 -0.50477891   0.0075352064  0.0090787308  0.0E+00  1.    'p'
1
'FAISCEAU'
 'SCALING'                                                                                                      3
 1  3
  MULTIPOL VKIC
  -1
  78.05648                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  78.05648
  1
  MULTIPOL QUAD
  -1
  78.05648
  1
 .............
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  0.2e3    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 .............
 'CAVITE'                                                                85                                     86
  2.1          .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365          9cavitiesx32kV, phi_s=180-30deg
```

```
from zgoubi.res :

     Particle  properties :

                     Mass          =    938.272        MeV/c2
                     Charge        =   1.602176E-19    C
                     G  factor     =    1.79285

              Reference  data :
                    rigidity (kG.cm)      :   78056.5
                    mass (MeV/c2)         :   938.272
                    momentum (MeV/c)      :   23400.7
                    energy, total (MeV)   :   23419.5
                    energy, kinetic (MeV) :   22481.3
                    beta = v/c            :  0.9991971305
                    gamma                 :   24.96029519
                    beta*gamma            :   24.94025534
```

#### Strength

From Figs. 131, 132 one gets

$p_{init} \approx 0.999755,\ \ p_{final} \approx 0.62962$

Eq. 2 yields

$A^2 = 0.2047056,$

$|J_n|^2 = 5.7466214E - 06$

$\hat{z}_{c.o.} = 0.275$ mm

$\gamma G = 45$ **(22.61211 GeV)**

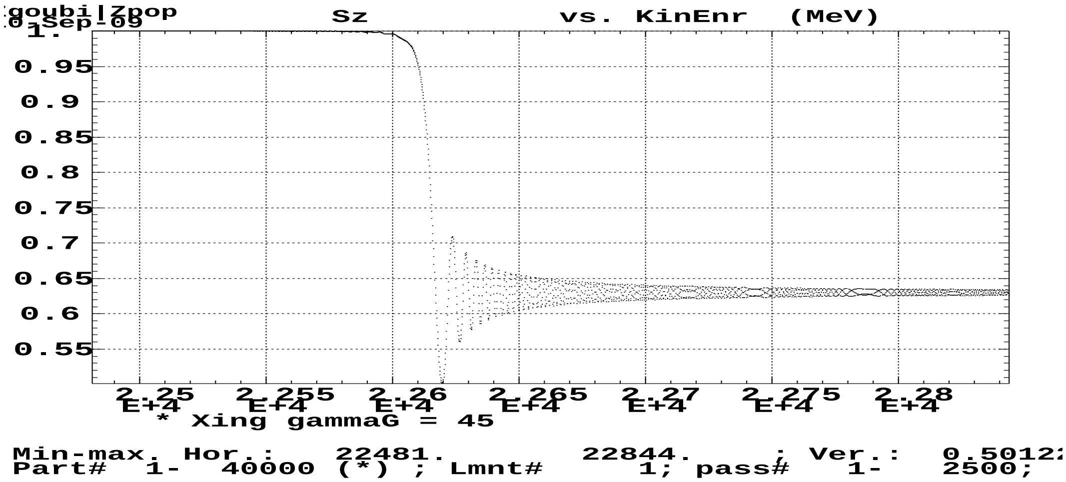



Figure 130: $S_z$ versus kinetic energy.

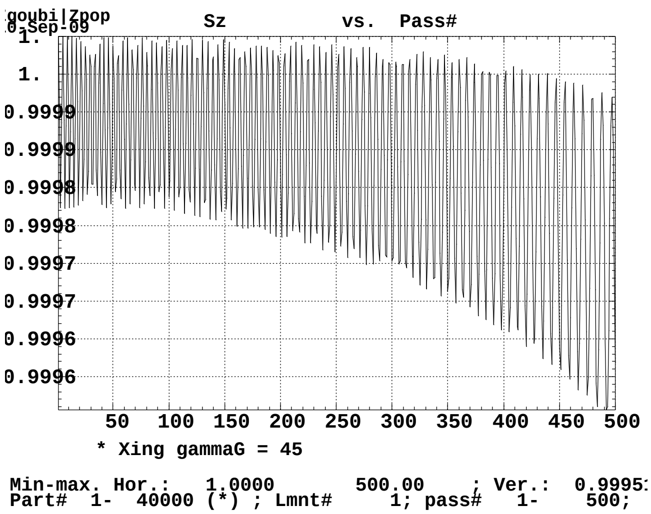



Figure 131: Zoom on initial $S_z$.

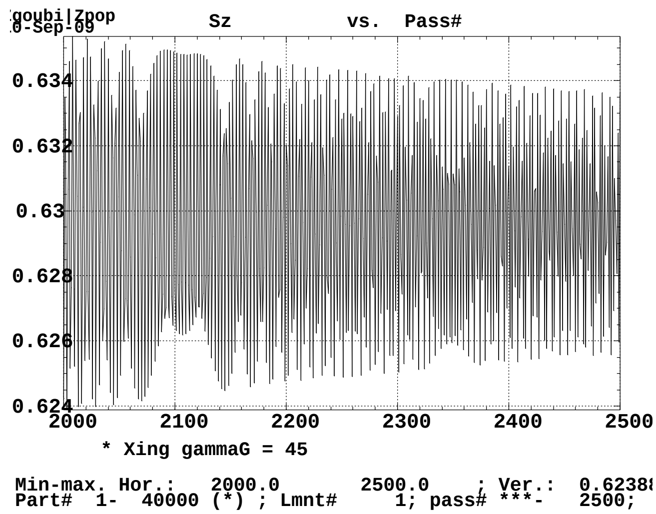



Figure 132: Zoom on final $S_z$.

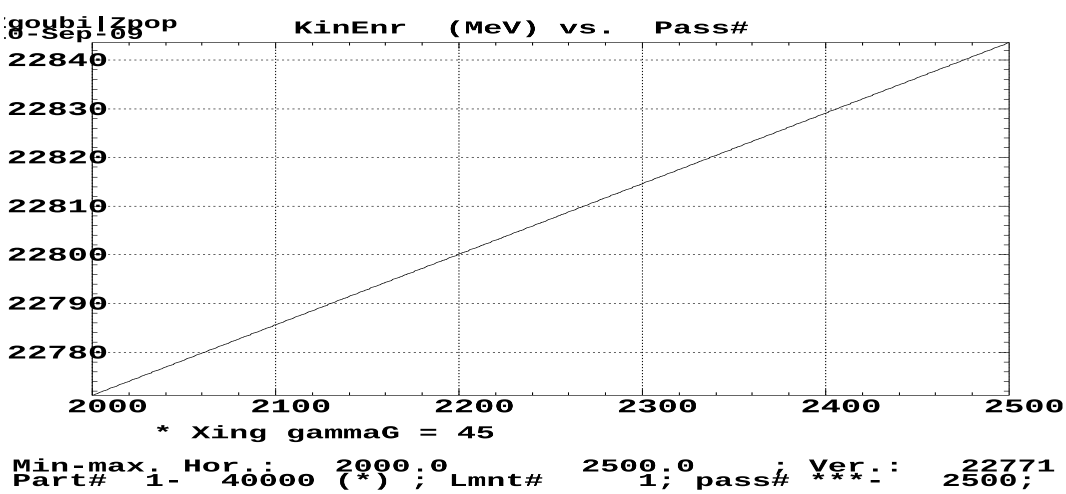



Figure 133: Kinetic E versus turn number.

$\gamma G = 45$ **(22.61211 GeV)**

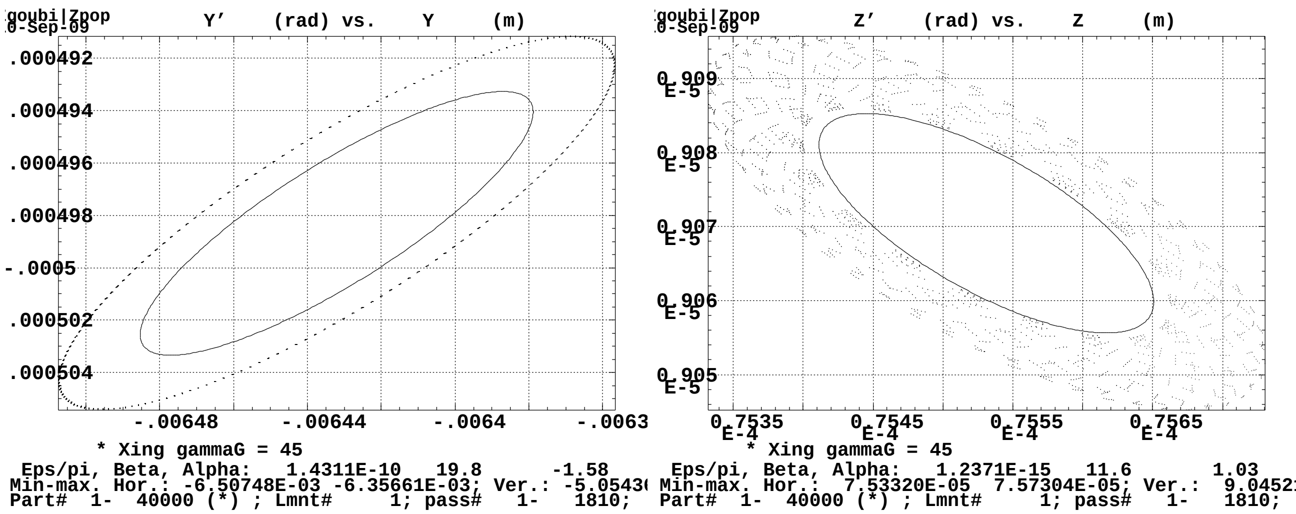



Figure 134: Left : x-x', right : z-z'.

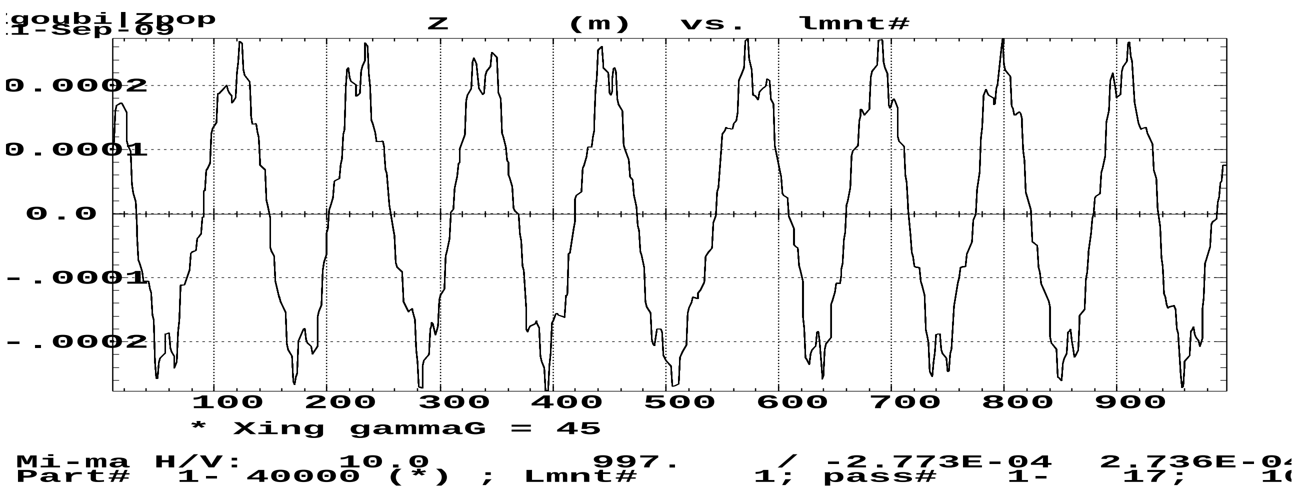



Figure 135: z closed orbit along the ring circumference (horizontal axis is pick-up number).

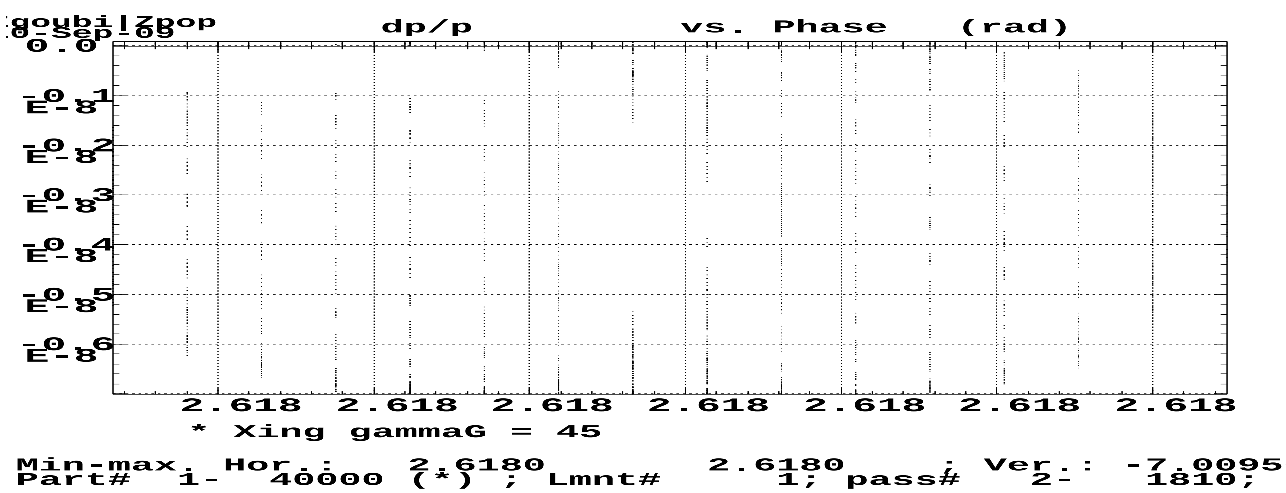



Figure 136: dp-phase.

# 5 Static neighboring of resonances

## 5.1 Intrinsic resonances

In static mode, at distance

$$\Delta = \text{ distance to the resonance } = \gamma G - (n \times M - \nu_z)$$

from an isolated spin resonance, the average value of the vertical component of an initially vertical $\vec{S}$ satisfies

$$\bar{S}_z^2 = \frac{1}{1 + |J_n|^2/\Delta^2}, \qquad \text{hence} \quad |J_n|^2 = \Delta^2 / \left(\frac{1}{1 - \bar{S}_z^2} - 1\right)$$

which yields $|J_n|$ from a measurement of $S_z(\Delta)$.

In order to perform a simulation of stable precession of $\vec{S}$ around a fixed axis, test particles are launched at various, fixed, energies $\gamma = (\Delta + (n \times M - \nu_z))/G$ (hence various $\Delta$), on non-zero vertical invariant $\epsilon_z$, on chromatic horizontal closed orbit corresponding to that energy and with initial $\vec{S} \equiv \vec{S}_z$ polarization.

The matching of the $z$-projection of the ray-traced $S_z$ component with $\bar{S}_z$, Eq. above, delivers best $\nu_z$ and $\epsilon$ values, these are displayed in Tab. 7 page 70.

### 5.1.1 $\gamma G = \nu_z$ (3.648013 GeV)

#### Strengths

Resonance strengths and vertical tunes are derived from the numerical simulation results by matching of numerical $\bar{S}_z(\gamma G)$ using Eq. 4, page 10.

This yields the results in Tab. 6.

Table 6: Dependence of resonance strength squared ($|J_n|^2$) on vertical invariant ($\epsilon_z$). $\nu_z$ compares fairly well with Fourier analysis of particle motion on $\epsilon_z$ invariant.

| $\epsilon_z/\pi$ ($10^{-6}$m.rad) | $\nu_z$ (Fourier) | $\nu_z$ (fit) | $\lvert J_n\rvert^2$ ($10^{-5}$) | $\lvert J_n\rvert^2/\epsilon_z/\pi$ |
|---|---|---|---|---|
| 0.00188 | | 8.7634 | 0.07406 | 390 |
| 0.0164 | | 8.7630 | 0.7458 | 455 |

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.0019\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

gG=nu. Data generated by searchCO
'OBJET'
14.97463e3
2
 18  1                         V emittance=0.0019e-6
 -1.646363E+00 -1.209158E+00 -7.000000E-03 -5.000000E-03  0.0E+00  9.95000000E-01 'p'      3626.04 MeV
 -1.148352E+00 -8.558551E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.97500000E-01 'p'      3637.03 MeV
 -7.447566E-01 -5.700890E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99500000E-01 'p'      3645.81 MeV
 -7.143164E-01 -5.485654E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99650000E-01 'p'      3646.47 MeV
 -6.940100E-01 -5.342098E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99750000E-01 'p'      3646.91 MeV
 -6.838529E-01 -5.270301E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99800000E-01 'p'      3647.13 MeV
 -6.736934E-01 -5.198490E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99850000E-01 'p'      3647.35 MeV
 -6.635312E-01 -5.126667E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99900000E-01 'p'      3647.57 MeV
 -6.533665E-01 -5.054831E-01 -7.000000E-03 -5.000000E-03  0.0E+00  9.99950000E-01 'p'      3647.79 MeV
 -6.431993E-01 -4.982983E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00000000E+00 'p'      3648.01 MeV
 -6.330295E-01 -4.911121E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00005000E+00 'p'      3648.23 MeV
 -6.228571E-01 -4.839247E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00010000E+00 'p'      3648.45 MeV
 -6.126822E-01 -4.767360E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00015000E+00 'p'      3648.67 MeV
 -6.025048E-01 -4.695461E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00020000E+00 'p'      3648.89 MeV
 -5.821424E-01 -4.551625E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00030000E+00 'p'      3649.33 MeV
 -5.515798E-01 -4.335778E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00045000E+00 'p'      3649.99 MeV
 -5.107943E-01 -4.047810E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00065000E+00 'p'      3650.87 MeV
 -4.597541E-01 -3.687560E-01 -7.000000E-03 -5.000000E-03  0.0E+00  1.00090000E+00 'p'      3651.97 MeV
      1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Checking computation results : for the 20 particles above, from 1200 turns analysis, their H closed orbits, fractional tunes (Fourier analysis) and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU      1-XNU    1-ZNU               epsilon/pi_X,Z,l               time(mus)  E(MeV)
 -1.646363E-02 -1.209158E-03  0.184293  0.243681  0.815707  0.756319  1.983219E-17  9.463248E-10  1.078902E-07   0.00000   3626.04
 -1.148352E-02 -8.558569E-04  0.236103  0.240276  0.763897  0.759724  6.203007E-18  9.437355E-10  7.235493E-08   1.37518   3637.03
 -7.447570E-03 -5.700879E-04  0.277672  0.237293  0.722328  0.762707  5.720399E-18  9.416535E-10  1.039773E-08   1.83348   3645.81
 -7.143173E-03 -5.485666E-04  0.280750  0.237144  0.719250  0.762856  4.351323E-18  9.414963E-10  2.902687E-08   2.06265   3646.47
 -6.940089E-03 -5.342087E-04  0.282798  0.236887  0.717202  0.763113  4.297505E-18  9.413920E-10  5.285475E-08   2.20016   3646.91
 -6.838527E-03 -5.270293E-04  0.283828  0.236814  0.716172  0.763186  5.418882E-18  9.413402E-10  4.505645E-08   2.29183   3647.13
 -6.736942E-03 -5.198491E-04  0.284866  0.236765  0.715134  0.763235  3.527709E-18  9.412885E-10  4.245698E-08   2.35730   3647.35
 -6.635313E-03 -5.126674E-04  0.285919  0.236722  0.714081  0.763278  4.063598E-18  9.412366E-10  3.422547E-08   2.40641   3647.57
 -6.533653E-03 -5.054814E-04  0.287004  0.236668  0.712996  0.763332  3.868776E-18  9.411843E-10  6.065265E-08   2.44460   3647.79
 -6.432005E-03 -4.982988E-04  0.288085  0.236581  0.711915  0.763419  3.994566E-18  9.411317E-10  9.531118E-08   2.47516   3648.01
 -6.330296E-03 -4.911123E-04  0.289138  0.236422  0.710862  0.763578  3.260158E-18  9.410789E-10  1.516312E-08   2.50016   3648.23
 -6.228566E-03 -4.839242E-04  0.290175  0.236333  0.709825  0.763667  3.709130E-18  9.410263E-10  5.502039E-08   2.52099   3648.45
 -6.126822E-03 -4.767353E-04  0.291205  0.236279  0.708795  0.763721  3.393409E-18  9.409740E-10  9.574401E-08   2.53861   3648.67
 -6.025061E-03 -4.695472E-04  0.292230  0.236235  0.707770  0.763765  3.275072E-18  9.409217E-10  4.678886E-08   2.55372   3648.89
 -5.821414E-03 -4.551620E-04  0.294278  0.236110  0.705722  0.763890  2.597808E-18  9.408175E-10  6.498435E-08   2.56681   3649.33
 -5.515801E-03 -4.335775E-04  0.297380  0.235790  0.702620  0.764210  2.040421E-18  9.406618E-10  7.018281E-08   2.57826   3649.99
 -5.107945E-03 -4.047817E-04  0.301655  0.235489  0.698345  0.764511  1.841409E-18  9.404511E-10  1.646254E-08   2.58835   3650.87
 -4.597556E-03 -3.687562E-04  0.306793  0.235137  0.693207  0.764863  1.077477E-17  9.401906E-10  3.335808E-08   2.59732   3651.97
```

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.0019\,10^{-6}$

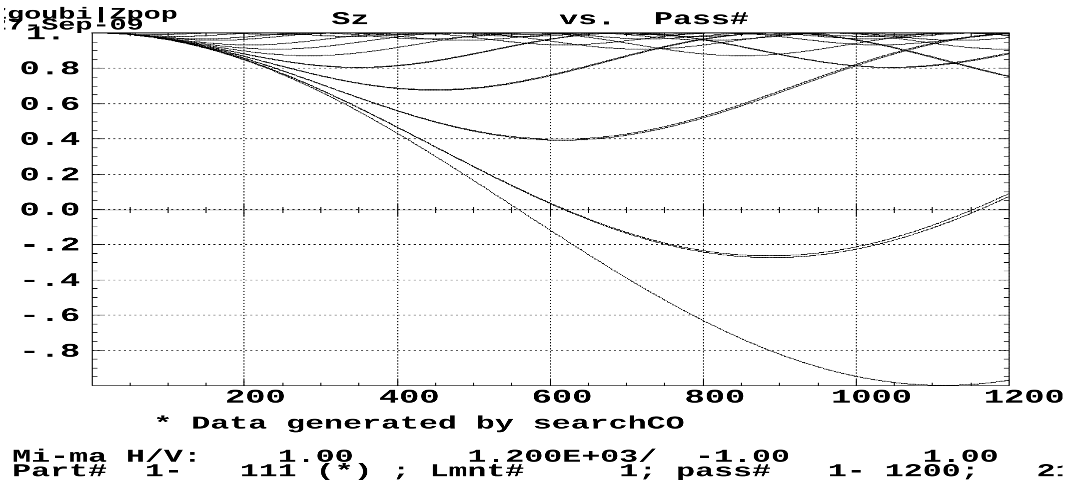



Figure 137: $S_z$ versus turn number for various distances to the resonance.

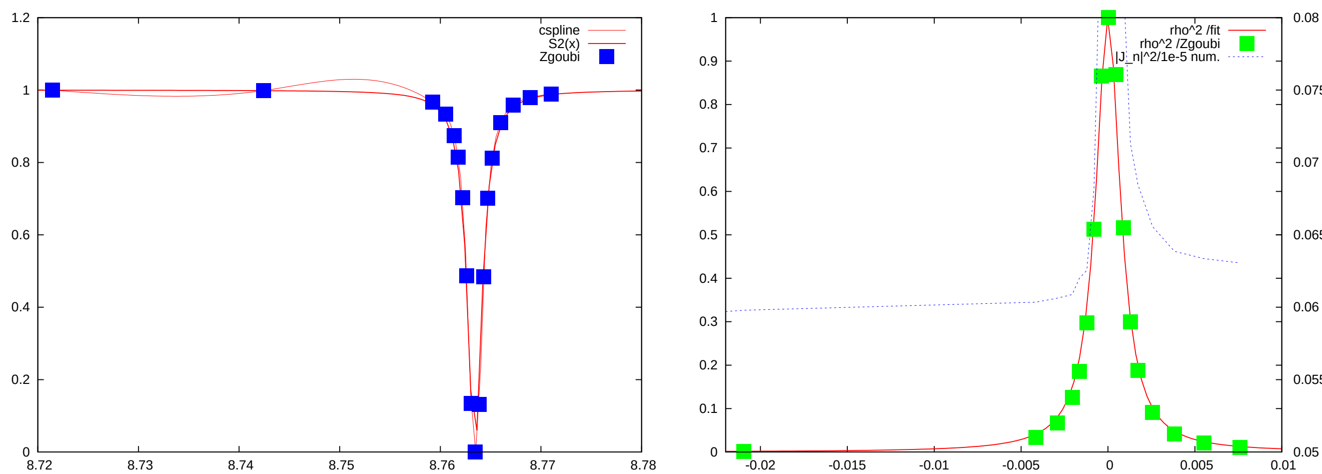



Figure 138: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

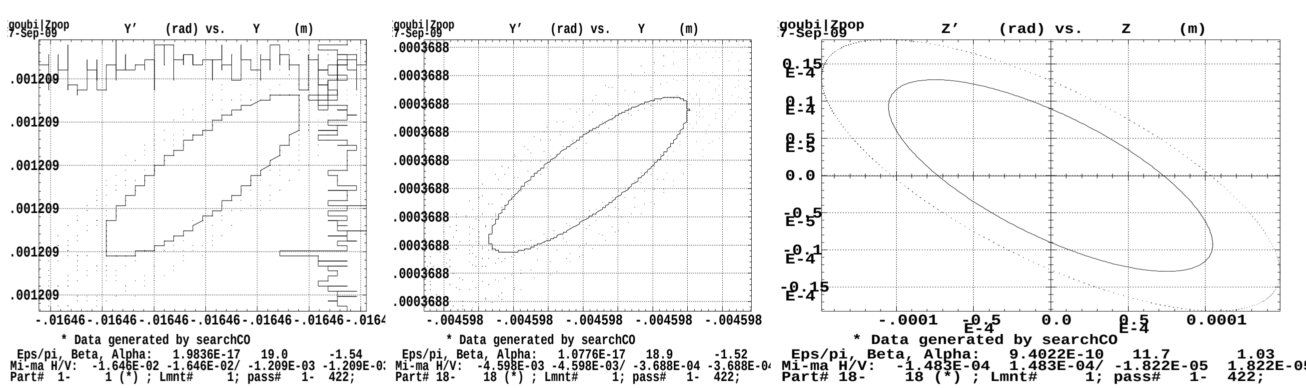



Figure 139: Left : x-x' of particles at min. and max. $p/p_0$. Right : z-z' (all particles superimposed).

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.017\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

gG=nu. Data generated by searchCO
'OBJET'
14.97463e3
2
 14  1
 -7.447527E-01 -5.700859E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99500000E-01 'p'      3645.81 MeV
 -7.143145E-01 -5.485637E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99650000E-01 'p'      3646.47 MeV
 -6.940190E-01 -5.342162E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99750000E-01 'p'      3646.91 MeV
 -6.838620E-01 -5.270365E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99800000E-01 'p'      3647.13 MeV
 -6.737025E-01 -5.198555E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99850000E-01 'p'      3647.35 MeV
 -6.635404E-01 -5.126732E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99900000E-01 'p'      3647.57 MeV
 -6.533757E-01 -5.054897E-01 -2.000000E-02 -1.570000E-02  0.0E+00  9.99950000E-01 'p'      3647.79 MeV
 -6.432085E-01 -4.983049E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00000000E+00 'p'      3648.01 MeV
 -6.330387E-01 -4.911187E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00005000E+00 'p'      3648.23 MeV
 -6.228664E-01 -4.839314E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00010000E+00 'p'      3648.45 MeV
 -6.126915E-01 -4.767427E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00015000E+00 'p'      3648.67 MeV
 -6.025141E-01 -4.695529E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00020000E+00 'p'      3648.89 MeV
 -5.821415E-01 -4.551617E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00030000E+00 'p'      3649.33 MeV
 -5.515787E-01 -4.335769E-01 -2.000000E-02 -1.570000E-02  0.0E+00  1.00045000E+00 'p'      3649.99 MeV
   1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
   'FAISCEAU'
    'SCALING'                                                                                            3
    1  2
     MULTIPOL SBEN
     -1
     14.97463
     1
     MULTIPOL QUAD
     -1
     14.97463
     1
    'PARTICUL'                                                                                           4
    938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
```

Checking computation results : for the 20 particles above, from 200 turns analysis, their H closed orbits, fractional tunes (Fourier analysis) and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU      1-XNU    1-ZNU                epsilon/pi_X,Z,l               time(mus)  E(MeV)
```

$\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.017\,10^{-6}$

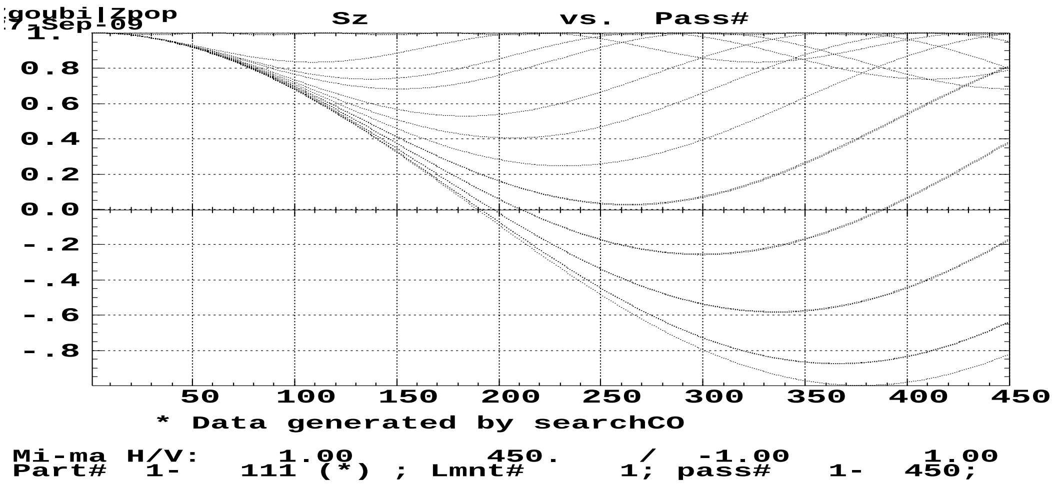



Figure 140: $S_z$ versus turn number for various distances to the resonance.

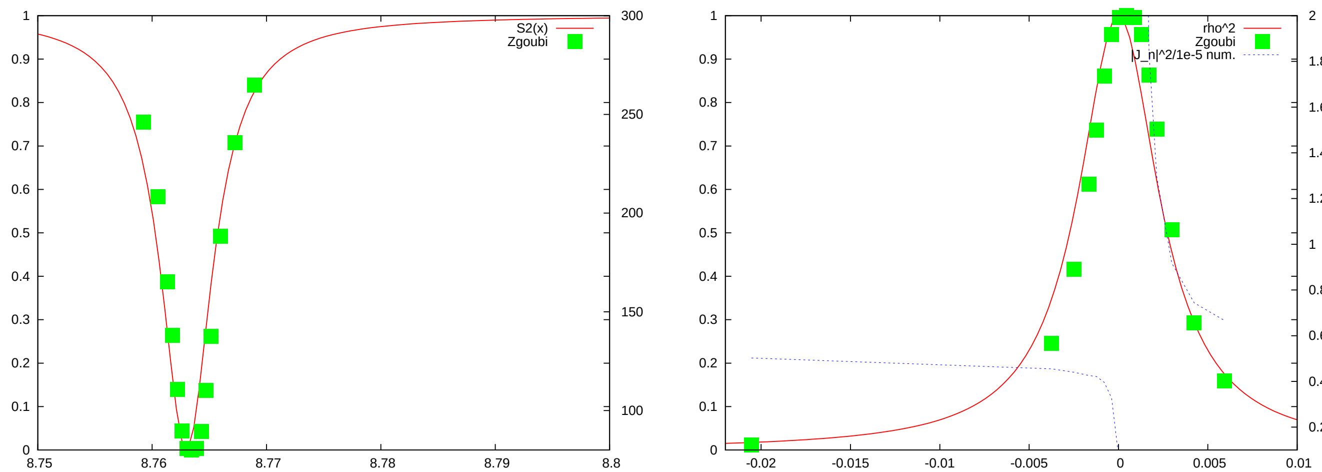



Figure 141: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

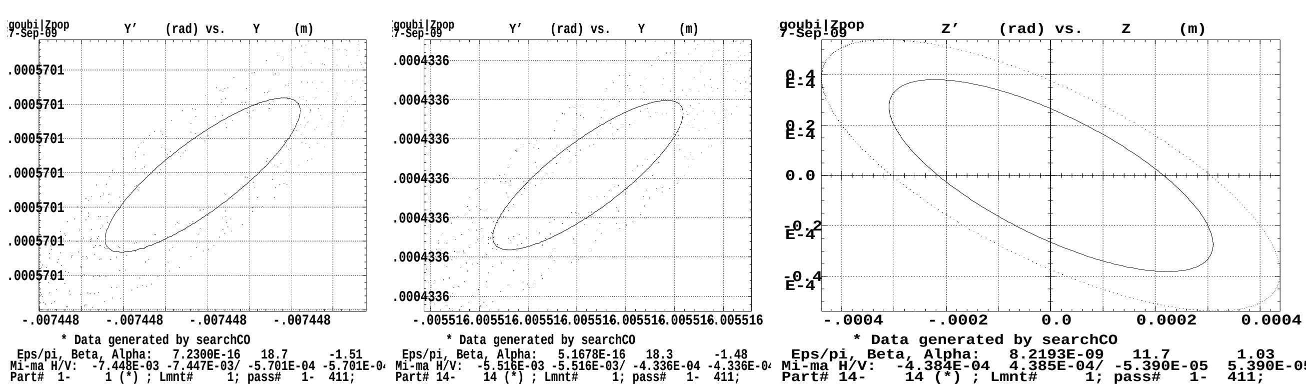



Figure 142: Left : x-x' of particles at min. and max. $p/p_0$. Right : z-z' (all particles superimposed).

### 5.1.2 $\gamma G = 48 - \nu_z$ (19.59585 GeV)

#### Strengths

Resonance strengths and vertical tunes are derived from the numerical simulation results by matching of numerical $\bar{S}_z(\gamma G)$ using Eq. 4, page 10.

This yields the results in Tab. 7. $\nu_z$ compares fairly well with Fourier analysis of particle motion on $\epsilon_z$ invariant. $|J_n|^2/\epsilon_z/\pi$ appears to be but weakly dependent on $\epsilon_z$ in agreement with the results of resonance crossing, Sec. 4.1.7 (Tab. 4 page 35).

Table 7: Dependence of resonance strength squared, $|J_n|^2$, on vertical invariant ($\epsilon_z$).

| $\epsilon_z/\pi$ ($10^{-6}$m.rad) | $\nu_z$ (Fourier) | $\nu_z$ (fit) | $\lvert J_n\rvert^2$ ($10^{-5}$) | $\lvert J_n\rvert^2/\epsilon_z/\pi$ |
|---|---|---|---|---|
| 0.125 | .774 | 8.7740 | 0.354 | 27 |
| 2 | .774 | 8.77 | 6.249 | 31 |

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.125\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

48-nu, static
Data generated by searchCO
'OBJET'
68.42292e3
2
 15  1                        V invariant=0.125e-6
     x            x'           z         z'          dl       p/p0
  5.238532E-01  3.225314E-01 0.121    -0.107   0.0E+00  9.99240000E-01 'p'     19580.28 MeV
  5.511955E-01  3.416643E-01 0.121    -0.107   0.0E+00  9.99370000E-01 'p'     19582.94 MeV
  5.785543E-01  3.608041E-01 0.121    -0.107   0.0E+00  9.99500000E-01 'p'     19585.61 MeV
  5.996108E-01  3.755316E-01 0.121    -0.107   0.0E+00  9.99600000E-01 'p'     19587.65 MeV
  6.122493E-01  3.843701E-01 0.121    -0.107   0.0E+00  9.99660000E-01 'p'     19588.88 MeV
  6.206770E-01  3.902633E-01 0.121    -0.107   0.0E+00  9.99700000E-01 'p'     19589.70 MeV
  6.248939E-01  3.932105E-01 0.121    -0.107   0.0E+00  9.99720000E-01 'p'     19590.11 MeV
  6.270709E-01  3.947346E-01 0.121    -0.107   0.0E+00  9.99730000E-01 'p'     19590.32 MeV
  6.291088E-01  3.961575E-01 0.121    -0.107   0.0E+00  9.99740000E-01 'p'     19590.52 MeV
  6.354291E-01  4.005778E-01 0.121    -0.107   0.0E+00  9.99770000E-01 'p'     19591.14 MeV
  6.438611E-01  4.064727E-01 0.121    -0.107   0.0E+00  9.99810000E-01 'p'     19591.96 MeV
  6.522946E-01  4.123683E-01 0.121    -0.107   0.0E+00  9.99850000E-01 'p'     19592.78 MeV
  6.586207E-01  4.167904E-01 0.121    -0.107   0.0E+00  9.99880000E-01 'p'     19593.39 MeV
  7.367155E-01  4.713593E-01 0.121    -0.107   0.0E+00  1.00025000E+00 'p'     19600.97 MeV
  8.107119E-01  5.230291E-01 0.121    -0.107   0.0E+00  1.00060000E+00 'p'     19608.15 MeV
   1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  1  1  1  1  1 1 1 1 1
 'FAISCEAU'
 'SCALING'
  1  2
   MULTIPOL SBEN
   -1
   67.98605
   1
   MULTIPOL QUAD
   -1
   67.98605
   1
  'PARTICUL'
  938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
  'SPNTRK'
   3
    'PICKUPS'
    1
    #Start
   'SPNSTORE'
    b_zgoubi.spn  #Start
      1
   'FAISTORE'
    b_zgoubi.fai  #End
      1
    'MARKER'  #Start
```

Checking computation results : for the 15 particles above, , from 400 turns analysis, their H closed orbits, fractional tunes and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU     1-XNU    1-ZNU            epsilon/pi_X,Z,l                time(mus)  E(MeV)
  5.238426E-03  3.225236E-04  0.452922 0.226474 0.547078 0.773526 4.944626E-15  6.260669E-08  8.490941E-08   2.69504  19580.3
  5.512045E-03  3.416666E-04  0.452434 0.226220 0.547566 0.773780 5.005877E-15  6.260760E-08  5.802153E-08   2.69505  19582.9
  5.785329E-03  3.607964E-04  0.451892 0.225955 0.548108 0.774045 5.086831E-15  6.260835E-08  1.556678E-08   2.69505  19585.6
  5.996270E-03  3.755278E-04  0.451525 0.225760 0.548475 0.774240 5.102903E-15  6.260872E-08  1.415163E-08   2.69506  19587.7
  6.122287E-03  3.843753E-04  0.451250 0.225647 0.548750 0.774353 5.165472E-15  6.260889E-08  3.396394E-08   2.69506  19588.9
  6.206834E-03  3.902411E-04  0.451102 0.225565 0.548898 0.774435 5.099527E-15  6.260900E-08  1.415165E-08   2.69506  19589.7
  6.248912E-03  3.932355E-04  0.451003 0.225520 0.548997 0.774480 4.766276E-15  6.260907E-08  9.906158E-09   2.69506  19590.1
  6.269512E-03  3.946532E-04  0.417172 0.225496 0.582828 0.774504 9.927906E-15  6.260913E-08  1.004768E-07   2.69506  19590.3
  6.291314E-03  3.961943E-04  0.450930 0.225473 0.549070 0.774527 4.806970E-15  6.260913E-08  4.953080E-08   2.69506  19590.5
  6.353840E-03  4.005324E-04  0.450822 0.225407 0.549178 0.774593 5.160008E-15  6.260924E-08  2.830333E-09   2.69506  19591.1
  6.439037E-03  4.065026E-04  0.450650 0.225327 0.549350 0.774673 5.214456E-15  6.260940E-08  4.953084E-08   2.69506  19592.0
  6.522486E-03  4.123469E-04  0.450513 0.225252 0.549487 0.774748 5.195458E-15  6.260958E-08  6.368255E-08   2.69506  19592.8
  6.586507E-03  4.167867E-04  0.450343 0.225196 0.549657 0.774804 5.240914E-15  6.260972E-08  5.236122E-08   2.69507  19593.4
  7.366901E-03  4.713678E-04  0.448848 0.224430 0.551152 0.775570 5.487470E-15  6.261195E-08  6.934356E-08   2.69508  19601.0
  8.107212E-03  5.230102E-04  0.447343 0.223718 0.552657 0.776282 6.407776E-15  6.261412E-08  4.670097E-08   2.69509  19608.1
```

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.125\,10^{-6}$

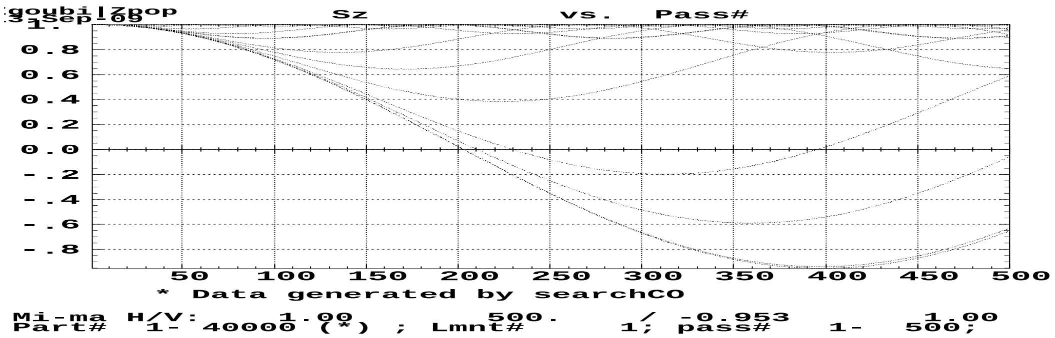



Figure 143: $S_z$ versus turn number for various distances to the resonance.

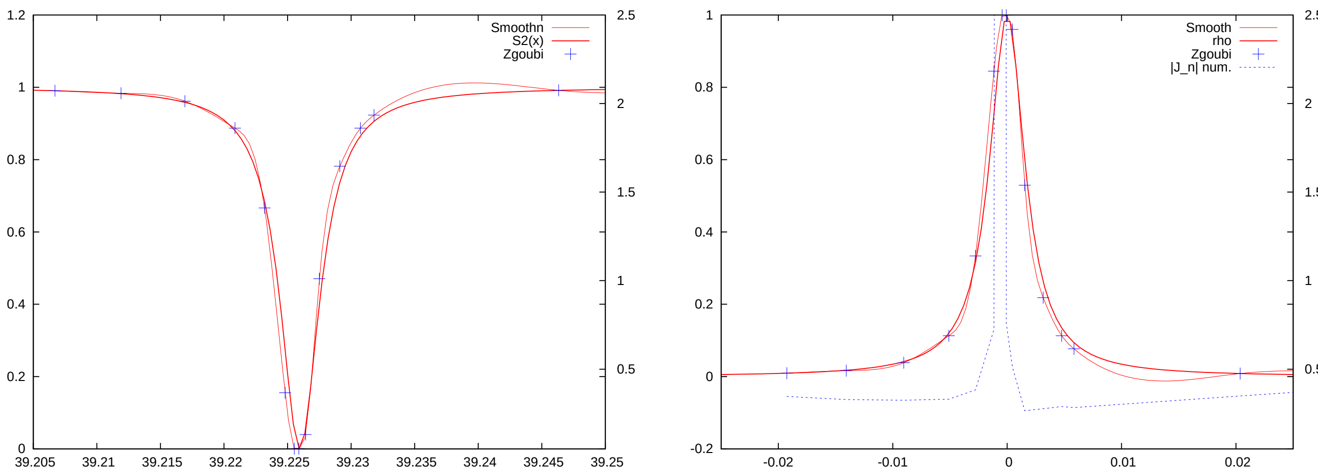



Figure 144: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

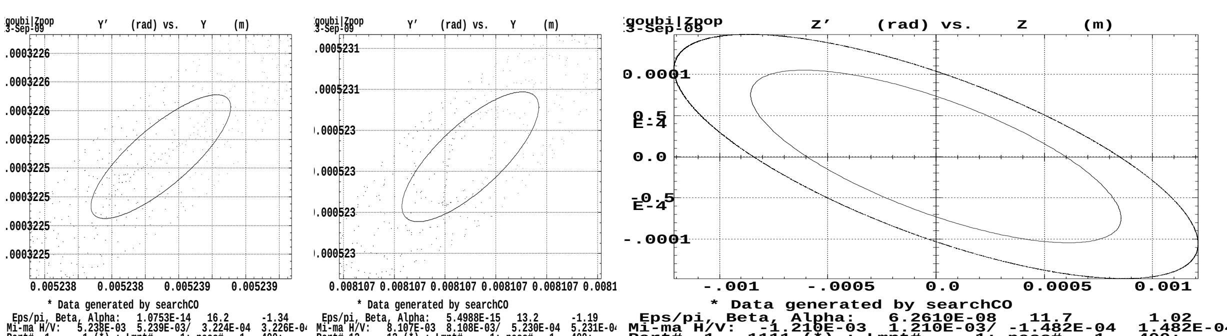



Figure 145: Left : x-x' of particles at min. and max. $p/p_0$. Right : z-z' (all particles superimposed).

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 2\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

48-nu, static
Data generated by searchCO
'OBJET'
68.42292e3
2
 17  1                        V invariant=1e-6
     x            x'            z         z'          dl       p/p0
  4.319392E-01  2.578809E-01  1.0E-02 -4.3E-01  0.0E+00  9.98800000E-01 'p'     19571.26 MeV
  4.739674E-01  2.872830E-01  1.0E-02 -4.3E-01  0.0E+00  9.99000000E-01 'p'     19575.36 MeV
  5.370833E-01  3.314176E-01  1.0E-02 -4.3E-01  0.0E+00  9.99300000E-01 'p'     19581.51 MeV
  5.792068E-01  3.608589E-01  1.0E-02 -4.3E-01  0.0E+00  9.99500000E-01 'p'     19585.61 MeV
  6.213739E-01  3.903209E-01  1.0E-02 -4.3E-01  0.0E+00  9.99700000E-01 'p'     19589.70 MeV
  6.361406E-01  4.006355E-01  1.0E-02 -4.3E-01  0.0E+00  9.99770000E-01 'p'     19591.14 MeV
  6.445808E-01  4.065304E-01  1.0E-02 -4.3E-01  0.0E+00  9.99810000E-01 'p'     19591.96 MeV
  6.488015E-01  4.094781E-01  1.0E-02 -4.3E-01  0.0E+00  9.99830000E-01 'p'     19592.37 MeV
  6.530225E-01  4.124260E-01  1.0E-02 -4.3E-01  0.0E+00  9.99850000E-01 'p'     19592.78 MeV
  6.593549E-01  4.168480E-01  1.0E-02 -4.3E-01  0.0E+00  9.99880000E-01 'p'     19593.39 MeV
  6.677993E-01  4.227447E-01  1.0E-02 -4.3E-01  0.0E+00  9.99920000E-01 'p'     19594.21 MeV
  6.846928E-01  4.345398E-01  1.0E-02 -4.3E-01  0.0E+00  1.00000000E+00 'p'     19595.85 MeV
  7.058183E-01  4.492871E-01  1.0E-02 -4.3E-01  0.0E+00  1.00010000E+00 'p'     19597.90 MeV
  7.480984E-01  4.787934E-01  1.0E-02 -4.3E-01  0.0E+00  1.00030000E+00 'p'     19602.00 MeV
  8.116001E-01  5.230863E-01  1.0E-02 -4.3E-01  0.0E+00  1.00060000E+00 'p'     19608.15 MeV
  1.237165E+00  8.192832E-01  1.0E-02 -4.3E-01  0.0E+00  1.00260000E+00 'p'     19649.13 MeV
  1.796276E+00  1.206708E+00  1.0E-02 -4.3E-01  0.0E+00  1.00520000E+00 'p'     19702.41 MeV
  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  1  1  1  1  1 1 1 1 1
 'FAISCEAU'
 'SCALING'
  1  2
   MULTIPOL SBEN
   -1
   67.98605
   1
   MULTIPOL QUAD
   -1
   67.98605
   1
  'PARTICUL'
  938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
  'SPNTRK'
   3
    'PICKUPS'
    1
    #Start
   'SPNSTORE'
    b_zgoubi.spn  #Start
      1
   'FAISTORE'
    b_zgoubi.fai  #End
      1
    'MARKER'  #Start
```

Checking computation results : for the 17 particles above, from 400 turns analysis, their H closed orbits, fractional tunes and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU     1-XNU    1-ZNU           epsilon/pi_X,Z,l                time(mus)  E(MeV)
  4.292329E-03  2.565613E-04  0.397391 0.227387 0.602609 0.772613 2.782303E-12  9.975757E-07  7.075746E-08   2.69503  19571.3
  4.727305E-03  2.860852E-04  0.401652 0.227005 0.598348 0.772995 2.813073E-12  9.975898E-07  5.660611E-09   2.69504  19575.4
  5.352981E-03  3.312343E-04  0.408019 0.226401 0.591981 0.773599 2.810842E-12  9.976181E-07  4.386990E-08   2.69505  19581.5
  5.768571E-03  3.588075E-04  0.412253 0.226013 0.587747 0.773987 2.777554E-12  9.976402E-07  1.556678E-08   2.69505  19585.6
  6.203395E-03  3.906452E-04  0.416524 0.225618 0.583476 0.774382 2.819153E-12  9.976524E-07  1.415165E-08   2.69506  19589.7
  6.331626E-03  3.984651E-04  0.418009 0.225463 0.581991 0.774537 2.803603E-12  9.976563E-07  2.830332E-09   2.69506  19591.1
  6.437418E-03  4.065558E-04  0.418836 0.225378 0.581164 0.774622 2.789871E-12  9.976590E-07  4.953084E-08   2.69506  19592.0
  6.460069E-03  4.079638E-04  0.419275 0.225338 0.580725 0.774662 2.770508E-12  9.976604E-07  4.103985E-08   2.69506  19592.4
  6.516286E-03  4.115880E-04  0.419715 0.225300 0.580285 0.774700 2.804960E-12  9.976619E-07  6.368254E-08   2.69506  19592.8
  6.572890E-03  4.162095E-04  0.420325 0.225244 0.579675 0.774756 2.806966E-12  9.976642E-07  5.236122E-08   2.69507  19593.4
  6.656351E-03  4.211746E-04  0.421203 0.225169 0.578797 0.774831 2.783820E-12  9.976677E-07  2.688821E-08   2.69507  19594.2
  6.832306E-03  4.344203E-04  0.422894 0.225002 0.577106 0.774998 2.783264E-12  9.976748E-07  7.075852E-09   2.69507  19595.9
  7.031513E-03  4.473885E-04  0.425016 0.224795 0.574984 0.775205 2.819403E-12  9.976841E-07  4.387033E-08   2.69507  19597.9
  7.469509E-03  4.789586E-04  0.429270 0.224377 0.570730 0.775623 2.732493E-12  9.977074E-07  1.556693E-08   2.69508  19602.0
  8.084513E-03  5.208150E-04  0.435641 0.223766 0.564359 0.776234 2.760451E-12  9.977392E-07  4.670097E-08   2.69509  19608.1
  1.234425E-02  8.178755E-04  0.478400 0.219459 0.521600 0.780541 2.889050E-12  9.979560E-07  3.396520E-08   2.69516  19649.1
  1.794068E-02  1.207013E-03  0.465735 0.213439 0.534265 0.786561 2.910560E-12  9.983185E-07  3.396634E-08   2.69525  19702.4
```

$\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 2\,10^{-6}$

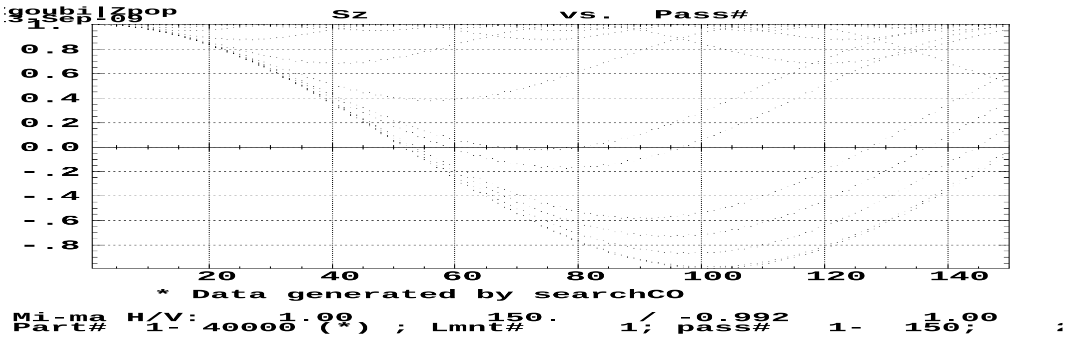



Figure 146: $S_z$ versus turn number for various distances to the resonance.

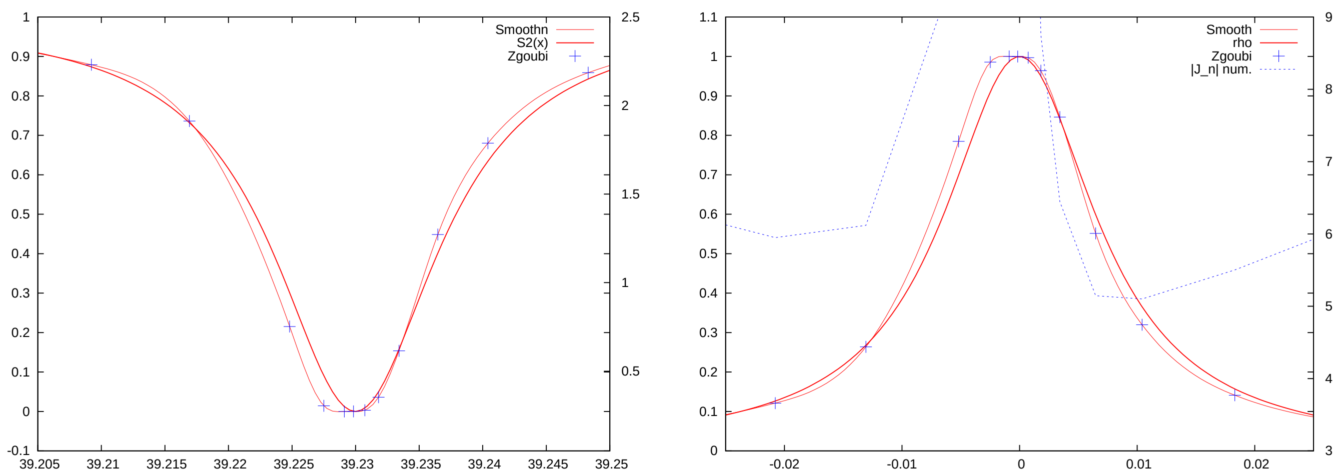



Figure 147: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

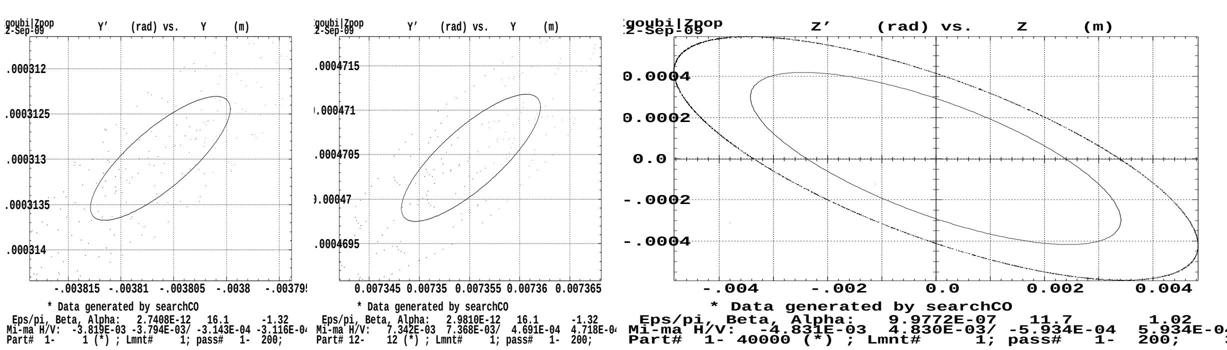



Figure 148: Left : x-x' of particles at min. and max. $p/p_0$.

### 5.1.3 $\gamma G = 36 + \nu_z$ (22.48832 GeV)

#### Strengths

Resonance strengths and vertical tunes are derived from the numerical simulation results by matching of numerical $\bar{S}_z(\gamma G)$ using Eq. 4, page 10.

This yields the results in Tab. 8.

Table 8: Dependence of resonance strength squared ($|J_n|^2$) on vertical invariant ($\epsilon_z$). $\nu_z$ compares fairly well with Fourier analysis of particle motion on $\epsilon_z$ invariant.

| $\epsilon_z/\pi$ ($10^{-6}$m.rad) | $\nu_z$ (Fourier) | $\nu_z$ (fit) | $\|J_n\|^2$ ($10^{-5}$) | $\|J_n\|^2/\epsilon_z/\pi$ |
|---|---|---|---|---|
| 0.0017 | .763 | 8.7631 | 0.80592 | 4700 |
| 0.17 | | 8.7639 | 94.5166 | 5559 |

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.0017\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

36+nu. Data generated by searchCO
'OBJET'
78.07999e3
2
 20  1
 -9.470844E-01 -7.132618E-01  -0.00643   -0.00498          0.0E+00  9.98500000E-01 'p'     22453.23 MeV
 -8.662617E-01 -6.560475E-01  -0.00643   -0.00498          0.0E+00  9.98900000E-01 'p'     22462.59 MeV
 -8.055577E-01 -6.130895E-01  -0.00643   -0.00498          0.0E+00  9.99200000E-01 'p'     22469.61 MeV
 -7.650340E-01 -5.844263E-01  -0.00643   -0.00498          0.0E+00  9.99400000E-01 'p'     22474.28 MeV
 -7.447550E-01 -5.700858E-01  -0.00643   -0.00498          0.0E+00  9.99500000E-01 'p'     22476.62 MeV
 -7.244648E-01 -5.557393E-01  -0.00643   -0.00498          0.0E+00  9.99600000E-01 'p'     22478.96 MeV
 -7.041638E-01 -5.413870E-01  -0.00643   -0.00498          0.0E+00  9.99700000E-01 'p'     22481.30 MeV
 -6.838522E-01 -5.270291E-01  -0.00643   -0.00498          0.0E+00  9.99800000E-01 'p'     22483.64 MeV
 -6.635304E-01 -5.126658E-01  -0.00643   -0.00498          0.0E+00  9.99900000E-01 'p'     22485.98 MeV
 -6.533656E-01 -5.054822E-01  -0.00643   -0.00498          0.0E+00  9.99950000E-01 'p'     22487.15 MeV
 -6.431983E-01 -4.982973E-01  -0.00643   -0.00498          0.0E+00  1.00000000E+00 'p'     22488.32 MeV
 -6.330285E-01 -4.911112E-01  -0.00643   -0.00498          0.0E+00  1.00005000E+00 'p'     22489.49 MeV
 -6.228561E-01 -4.839238E-01  -0.00643   -0.00498          0.0E+00  1.00010000E+00 'p'     22490.66 MeV
 -6.025037E-01 -4.695452E-01  -0.00643   -0.00498          0.0E+00  1.00020000E+00 'p'     22493.00 MeV
 -5.821413E-01 -4.551616E-01  -0.00643   -0.00498          0.0E+00  1.00030000E+00 'p'     22495.33 MeV
 -5.617687E-01 -4.407730E-01  -0.00643   -0.00498          0.0E+00  1.00040000E+00 'p'     22497.67 MeV
 -5.413860E-01 -4.263795E-01  -0.00643   -0.00498          0.0E+00  1.00050000E+00 'p'     22500.01 MeV
 -5.209933E-01 -4.119812E-01  -0.00643   -0.00498          0.0E+00  1.00060000E+00 'p'     22502.35 MeV
 -4.597547E-01 -3.687569E-01  -0.00643   -0.00498          0.0E+00  1.00090000E+00 'p'     22509.37 MeV
 -3.779636E-01 -3.110579E-01  -0.00643   -0.00498          0.0E+00  1.00130000E+00 'p'     22518.72 MeV
 1 1 1 1 1 1 1 1 1 1  1 1 1 1 1 1 1 1 1 1
  'SCALING'                                                                                                    3
  1  2
   MULTIPOL SBEN
   -1
   78.07999
   1
   MULTIPOL QUAD
   -1
   78.07999
   1
  'PARTICUL'                                                                                                   4
  938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
  'SPNTRK'                                                                                                     5
   3
```

Checking computation results : for the 20 particles above, from 200 turns analysis, their H closed orbits, fractional tunes (Fourier analysis) and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU      1-XNU    1-ZNU               epsilon/pi_X,Z,l                 time(mus)  E(MeV)
 -9.470872E-03 -7.132596E-04  0.256813  0.238834  0.743187  0.761166  1.324362E-16  8.428865E-10  5.941624E-08  2.69412   22453.2
 -8.663059E-03 -6.560778E-04  0.265148  0.238243  0.734852  0.761757  1.321423E-15  8.425258E-10  1.188331E-07  2.69413   22462.6
 -8.055589E-03 -6.130968E-04  0.271368  0.237784  0.728632  0.762216  2.727146E-16  8.422477E-10  8.318346E-08  2.69414   22469.6
 -7.650366E-03 -5.844278E-04  0.275550  0.237473  0.724450  0.762527  6.013444E-17  8.420647E-10  1.222291E-07  2.69415   22474.3
 -7.447593E-03 -5.700916E-04  0.277636  0.237315  0.722364  0.762685  2.244300E-17  8.419734E-10  2.376679E-08  2.69415   22476.6
 -7.244645E-03 -5.557404E-04  0.279712  0.237178  0.720288  0.762822  6.589209E-18  8.418826E-10  8.148624E-08  2.69416   22479.0
 -7.041638E-03 -5.413877E-04  0.281785  0.237017  0.718215  0.762983  2.089928E-18  8.417926E-10  4.583607E-08  2.69416   22481.3
 -6.838523E-03 -5.270286E-04  0.283866  0.236846  0.716134  0.763154  2.672892E-18  8.417030E-10  8.997461E-08  2.69416   22483.6
 -6.635325E-03 -5.126677E-04  0.285951  0.236703  0.714049  0.763297  3.347435E-18  8.416126E-10  6.960309E-08  2.69417   22486.0
 -6.533659E-03 -5.054828E-04  0.287002  0.236629  0.712998  0.763371  4.217753E-18  8.415669E-10  8.997478E-08  2.69417   22487.1
 -6.431985E-03 -4.982972E-04  0.288054  0.236546  0.711946  0.763454  4.480320E-18  8.415207E-10  1.154394E-07  2.69417   22488.3
 -6.330300E-03 -4.911119E-04  0.289100  0.236452  0.710900  0.763548  4.692319E-18  8.414741E-10  4.244099E-08  2.69417   22489.5
 -6.228580E-03 -4.839258E-04  0.290140  0.236369  0.709860  0.763631  5.298798E-18  8.414274E-10  6.111506E-08  2.69417   22490.7
 -6.025030E-03 -4.695446E-04  0.292219  0.236225  0.707781  0.763775  6.141337E-18  8.413342E-10  4.413871E-08  2.69418   22493.0
 -5.821433E-03 -4.551624E-04  0.294294  0.236072  0.705706  0.763928  5.324875E-18  8.412417E-10  1.544857E-07  2.69418   22495.3
 -5.617702E-03 -4.407750E-04  0.296376  0.235887  0.703624  0.764113  5.242630E-18  8.411499E-10  6.960352E-08  2.69418   22497.7
 -5.413859E-03 -4.263791E-04  0.298467  0.235738  0.701533  0.764262  5.434621E-18  8.410583E-10  7.130126E-08  2.69419   22500.0
 -5.209948E-03 -4.119820E-04  0.300571  0.235587  0.699429  0.764413  5.409531E-18  8.409670E-10  1.341144E-07  2.69419   22502.4
 -4.597559E-03 -3.687581E-04  0.306818  0.235096  0.693182  0.764904  6.153343E-18  8.406965E-10  1.018594E-08  2.69420   22509.4
 -3.779643E-03 -3.110580E-04  0.315191  0.234410  0.684809  0.765590  5.334159E-18  8.403249E-10  8.318563E-08  2.69421   22518.7
```

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.0017\,10^{-6}$

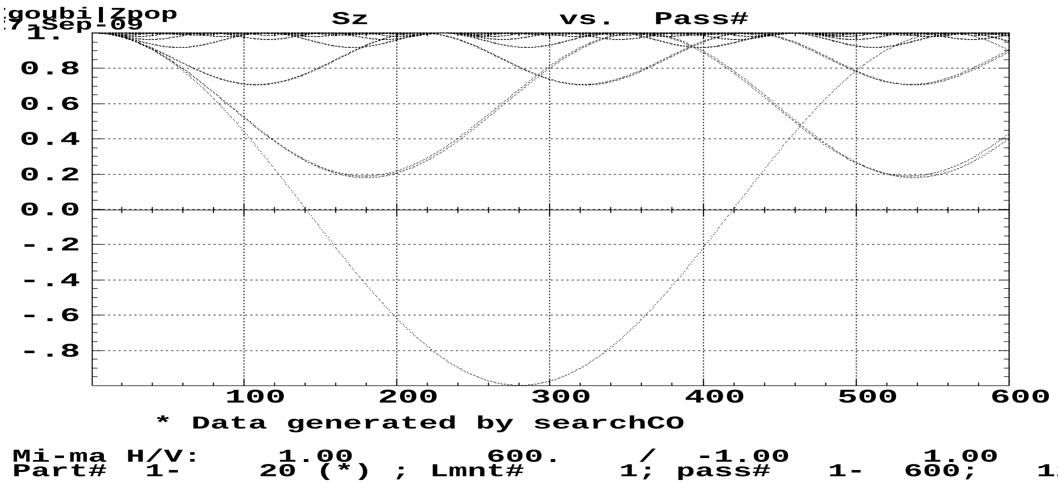



Figure 149: $S_z$ versus turn number for various distances to the resonance.

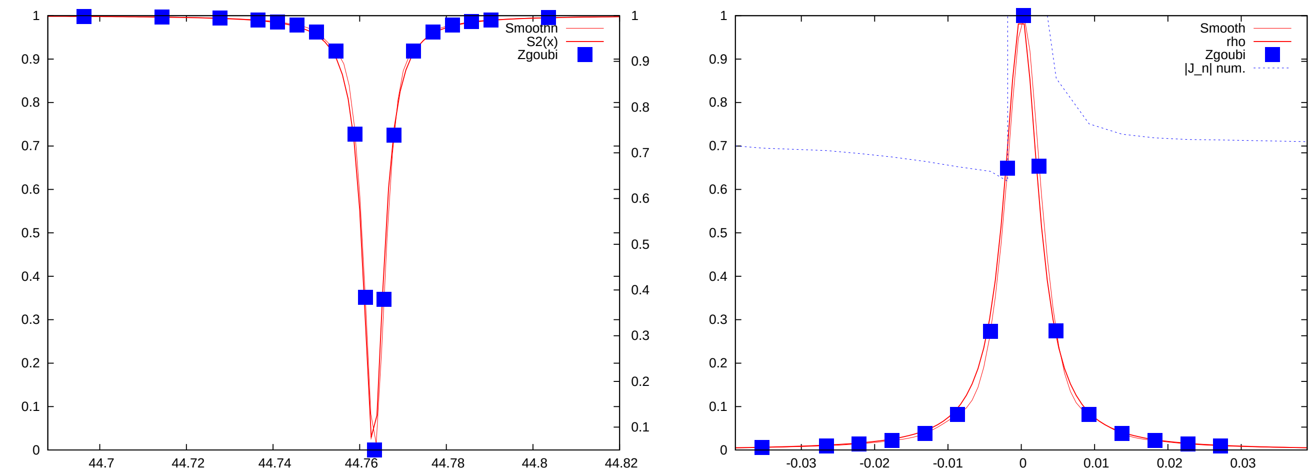



Figure 150: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

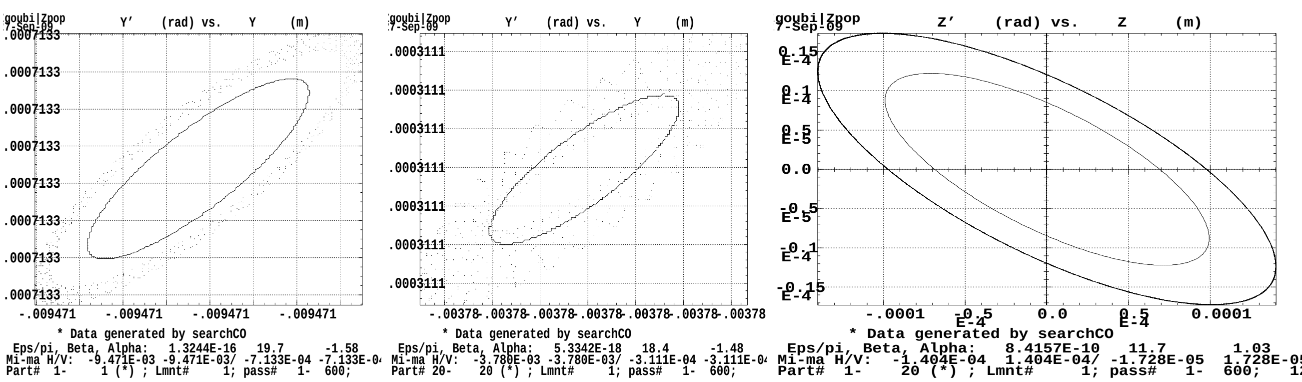



Figure 151: Left : x-x' of particles at min. and max. $p/p_0$. Right : z-z' (all particles superimposed).

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.17\,10^{-6}$

## Tracking data

```
Zgoubi.dat, excerpts :

36+nu, static
'OBJET'
78.07999e3
2
 20  1                        V invariant=0.17e-6
 -9.470844E-01 -7.132618E-01  -0.0643   -0.0498          0.0E+00  9.98500000E-01 'p'     22453.23 MeV
 -8.662617E-01 -6.560475E-01  -0.0643   -0.0498          0.0E+00  9.98900000E-01 'p'     22462.59 MeV
 -8.055577E-01 -6.130895E-01  -0.0643   -0.0498          0.0E+00  9.99200000E-01 'p'     22469.61 MeV
 -7.650340E-01 -5.844263E-01  -0.0643   -0.0498          0.0E+00  9.99400000E-01 'p'     22474.28 MeV
 -7.447550E-01 -5.700858E-01  -0.0643   -0.0498          0.0E+00  9.99500000E-01 'p'     22476.62 MeV
 -7.244648E-01 -5.557393E-01  -0.0643   -0.0498          0.0E+00  9.99600000E-01 'p'     22478.96 MeV
 -7.041638E-01 -5.413870E-01  -0.0643   -0.0498          0.0E+00  9.99700000E-01 'p'     22481.30 MeV
 -6.838522E-01 -5.270291E-01  -0.0643   -0.0498          0.0E+00  9.99800000E-01 'p'     22483.64 MeV
 -6.635304E-01 -5.126658E-01  -0.0643   -0.0498          0.0E+00  9.99900000E-01 'p'     22485.98 MeV
 -6.533656E-01 -5.054822E-01  -0.0643   -0.0498          0.0E+00  9.99950000E-01 'p'     22487.15 MeV
 -6.431983E-01 -4.982973E-01  -0.0643   -0.0498          0.0E+00  1.00000000E+00 'p'     22488.32 MeV
 -6.330285E-01 -4.911112E-01  -0.0643   -0.0498          0.0E+00  1.00005000E+00 'p'     22489.49 MeV
 -6.228561E-01 -4.839238E-01  -0.0643   -0.0498          0.0E+00  1.00010000E+00 'p'     22490.66 MeV
 -6.025037E-01 -4.695452E-01  -0.0643   -0.0498          0.0E+00  1.00020000E+00 'p'     22493.00 MeV
 -5.821413E-01 -4.551616E-01  -0.0643   -0.0498          0.0E+00  1.00030000E+00 'p'     22495.33 MeV
 -5.617687E-01 -4.407730E-01  -0.0643   -0.0498          0.0E+00  1.00040000E+00 'p'     22497.67 MeV
 -5.413860E-01 -4.263795E-01  -0.0643   -0.0498          0.0E+00  1.00050000E+00 'p'     22500.01 MeV
 -5.209933E-01 -4.119812E-01  -0.0643   -0.0498          0.0E+00  1.00060000E+00 'p'     22502.35 MeV
 -4.597547E-01 -3.687569E-01  -0.0643   -0.0498          0.0E+00  1.00090000E+00 'p'     22509.37 MeV
 -3.779636E-01 -3.110579E-01  -0.0643   -0.0498          0.0E+00  1.00130000E+00 'p'     22518.72 MeV
 1 1 1 1 1 1 1 1 1 1  1 1 1 1 1 1 1 1 1 1
  'SCALING'                                                                                                 3
  1  2
   MULTIPOL SBEN
   -1
   78.07999
   1
   MULTIPOL QUAD
   -1
   78.07999
   1
  'PARTICUL'                                                                                                4
  938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
```

Checking computation results : for the 20 particles above, from 200 turns analysis, their H closed orbits, fractional tunes (Fourier analysis) and complements to 1, H and V invariant values, energy :

```
%    XCO           X'CO          XNU      ZNU     1-XNU    1-ZNU           epsilon/pi_X,Z,l                     time(mus)  E(MeV)
 -9.471191E-03 -7.132492E-04  0.256831  0.238846  0.743169  0.761154  5.925320E-14  8.428749E-08  1.697588E-09   2.69412   22453.2
 -8.665436E-03 -6.562084E-04  0.265155  0.238248  0.734845  0.761752  7.102949E-14  8.424973E-08  1.188318E-08   2.69413   22462.6
 -8.056784E-03 -6.132896E-04  0.271387  0.237779  0.728613  0.762221  5.860524E-14  8.422212E-08  7.922146E-09   2.69414   22469.6
 -7.649985E-03 -5.843489E-04  0.275548  0.237463  0.724452  0.762537  5.496437E-14  8.420384E-08  1.075151E-08   2.69415   22474.3
 -7.449236E-03 -5.701867E-04  0.277640  0.237307  0.722360  0.762693  5.424775E-14  8.419471E-08  6.790436E-09   2.69415   22476.6
 -7.246618E-03 -5.559006E-04  0.279730  0.237154  0.720270  0.762846  5.443044E-14  8.418559E-08  1.641024E-08   2.69416   22479.0
 -7.041855E-03 -5.414722E-04  0.281801  0.236998  0.718199  0.763002  5.458121E-14  8.417649E-08  6.790453E-09   2.69416   22481.3
 -6.838782E-03 -5.269547E-04  0.283885  0.236844  0.716115  0.763156  5.488327E-14  8.416743E-08  9.619820E-09   2.69416   22483.6
 -6.638039E-03 -5.128970E-04  0.285944  0.236693  0.714056  0.763307  5.496553E-14  8.415843E-08  4.526980E-09   2.69417   22486.0
 -6.534031E-03 -5.055584E-04  0.287019  0.236616  0.712981  0.763384  5.491074E-14  8.415396E-08  1.923968E-08   2.69417   22487.1
 -6.432217E-03 -4.982846E-04  0.288055  0.236539  0.711945  0.763461  5.505559E-14  8.414952E-08  3.961112E-09   2.69417   22488.3
 -6.332021E-03 -4.911956E-04  0.289102  0.236460  0.710898  0.763540  5.518761E-14  8.414509E-08  5.658735E-09   2.69417   22489.5
 -6.230466E-03 -4.841332E-04  0.290139  0.236382  0.709861  0.763618  5.511605E-14  8.414069E-08  1.697622E-09   2.69417   22490.7
 -6.024385E-03 -4.694925E-04  0.292236  0.236227  0.707764  0.763773  5.515216E-14  8.413197E-08  1.188337E-08   2.69418   22493.0
 -5.823444E-03 -4.552429E-04  0.294313  0.236069  0.705687  0.763931  5.520688E-14  8.412332E-08  2.829376E-09   2.69418   22495.3
 -5.619226E-03 -4.409711E-04  0.296394  0.235905  0.703606  0.764095  5.503099E-14  8.411473E-08  4.527008E-09   2.69418   22497.7
 -5.413718E-03 -4.263393E-04  0.298460  0.235742  0.701540  0.764258  5.531948E-14  8.410615E-08  5.092890E-09   2.69419   22500.0
 -5.211530E-03 -4.120730E-04  0.300571  0.235577  0.699429  0.764423  5.505439E-14  8.409755E-08  7.922284E-09   2.69419   22502.4
 -4.598617E-03 -3.688727E-04  0.306842  0.235073  0.693158  0.764927  5.526524E-14  8.407108E-08  3.395277E-09   2.69420   22509.4
 -3.780573E-03 -3.110925E-04  0.315188  0.234408  0.684812  0.765592  5.530870E-14  8.403336E-08  1.697647E-08   2.69421   22518.7
```

$\gamma G = 36 + \nu_z$ (22.48832 GeV) - $\epsilon_z/\pi = 0.17\,10^{-6}$

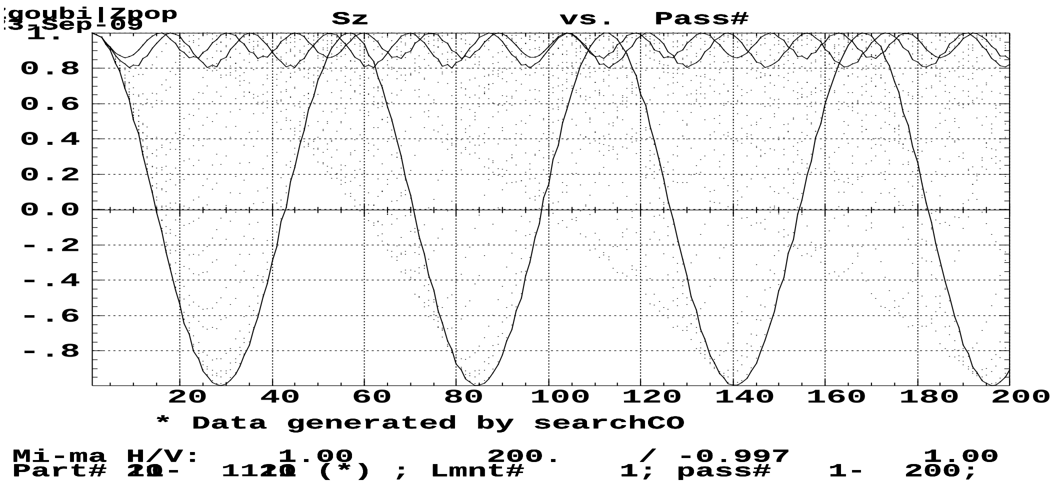



Figure 152: $S_z$ versus turn number for various distances to the resonance.

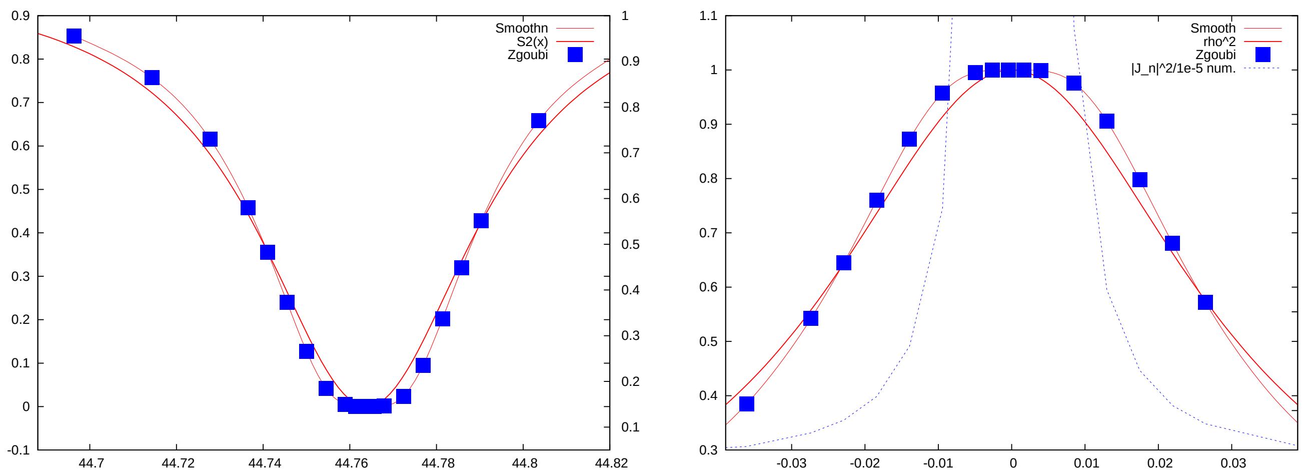



Figure 153: Matching $S_z^2(\Delta)$ (left, Eq. 4) and $\rho^2(\Delta)$ (right, Eq. 5) The right plot also shows $|J_n|^2$ from numerical data (with expectable lack of accuracy in the region $\Delta \to 0$).

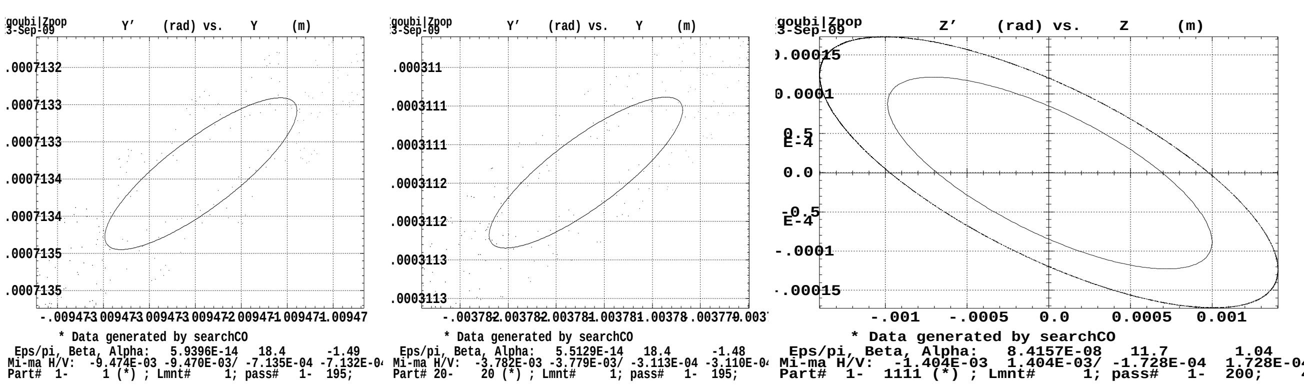



Figure 154: Left : x-x' of particles at min. and max. $p/p_0$. Right : z-z' (all particles superimposed).

# 6 Fresnel integrals approximation of weak resonances

The goal of this section is to check zgoubi against the weak resonance approximation of resonance crossing dynamics, see Sec. 2.3.4.

Matching the tracking results by the Fresnel integral approximation would yield the crossing speed $\alpha$ and the resonance strength $|J_n|$. In the figures below crossing velocity is taken to be the same as used earlier, $\alpha = 4.4096356\,10^{-5}$, and resonance strength is taken from Tabs. 2, 3, Froissard-Stora crossing conditions, whereas $\nu_z = 8.76346$ from Zgoubi, as earlier.

• $\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.002\,10^{-6}$

$\pi|J_n|^2/\alpha \approx 0.014$, higher order terms have negligible effect.

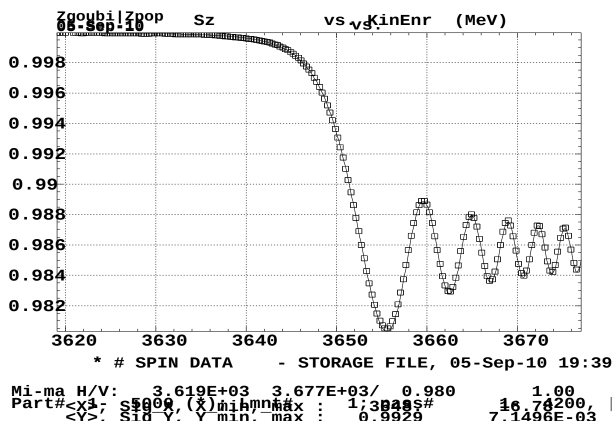



Figure 155: Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line), $\epsilon_z/\pi = 0.002\,10^{-6}$.

•$\theta < 0$ :

$$\left(\frac{p(\theta)}{p_i}\right)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2\left[(0.5 - C(-\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 - S(-\theta\sqrt{\frac{a}{\pi}}))^2\right]$$

• $\theta > 0$ :

$$\left(\frac{p(\theta)}{p_i}\right)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2\left[(0.5 + C(\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 + S(\theta\sqrt{\frac{a}{\pi}}))^2\right]$$

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)\,dt,\ S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)\,dt$$

• $\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.01\,10^{-6}$

$\pi|J_n|^2/\alpha \approx 0.071$, , higher order terms have sensible effect.

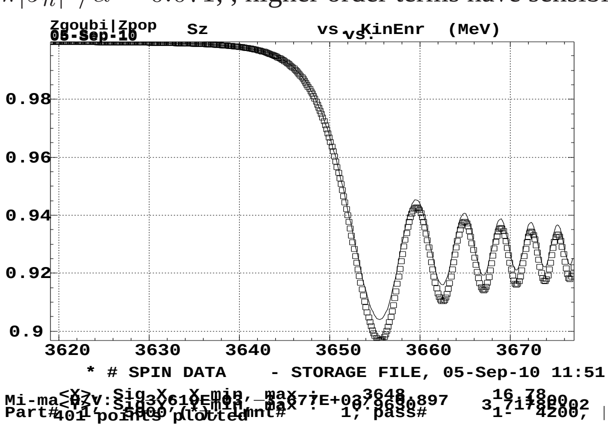



Figure 156: Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line) in a case of greater effect compared to the previous Fig. 155 : $\epsilon_z/\pi = 0.01\,10^{-6}$. The truncation of Froissard-Stora formula leads to an overestimate of $p_f/p_i$ (Eq. 7).

$2\exp(-A^2) - 1 \approx 1 - A^2$ no longer holds

• $\gamma G = 36 + \nu_z$ (22.4883 GeV) - $\epsilon_z/\pi = 0.0001\,10^{-6}$

$\pi |J_n|^2/\alpha \approx 0.013$, higher order terms have negligible effect.

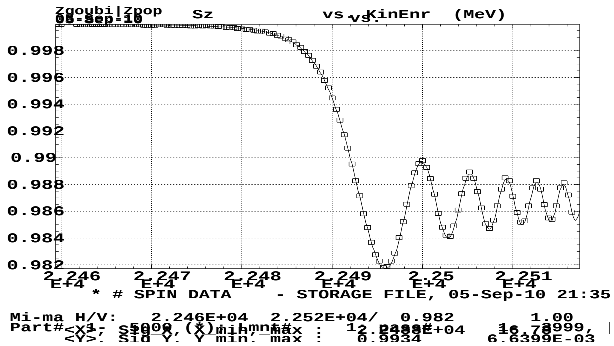



Figure 157: Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line) $\epsilon_z/\pi = 0.0001\,10^{-6}$.

• $\gamma G = 9$ (3.7718 GeV) - $z_{CO} = 1.3$ mm

$\pi |J_n|^2/\alpha \approx 0.0059$, higher order terms have negligible effect.

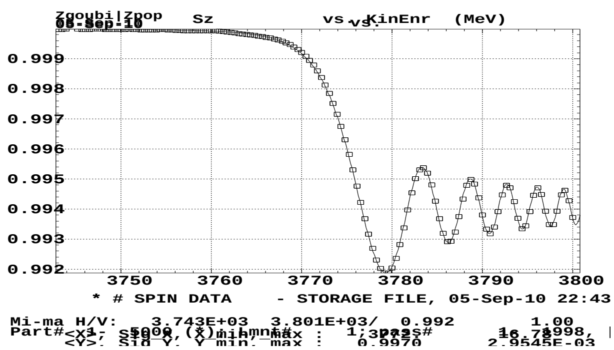



Figure 158: Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line).

• $\gamma G = 45$ (22.61211 GeV) - $z_{CO} = 0.028$ mm

$\pi |J_n|^2/\alpha \approx 0.004$, higher order terms have negligible effect.

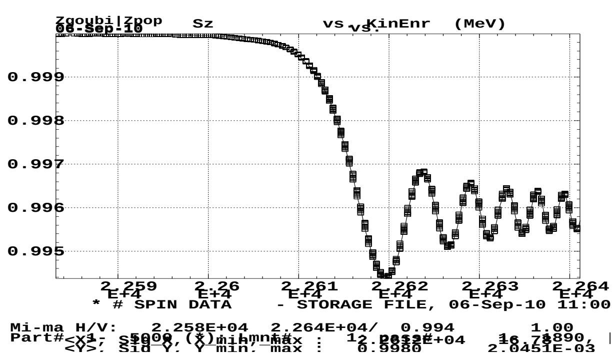



Figure 159: Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line).

# 7 $\vec{n}_0$ spin vector, AGS bare lattice

To perform this experiment, we consider a single particle at fixed energy, we sit at $T = 3775.00$ GeV within the span of $\gamma G = 9$ (T = 3.7716 GeV). The resonance is excited by a $\hat{z} \approx 2.8$ mm vertical closed orbit defect (same as earlier, Fig. 100 page 49).

The particle is launched on closed H and V orbit ($\epsilon_x = \epsilon_z = 0$, Fig. 160) with first $S_z = 1$, the polarization vector then oscillates around the $\vec{n}_0$ vector, Fig. 161.

The FIT procedure allows finding $\vec{n}_0$, typical zgoubi.dat input in App. B.2.

Given the FIT results, the particle is then launched, again on closed H and V orbit, with now $\vec{S} = closed - orbit\ \vec{S}$. Fig. 162 shows that the polarization vector, as observed turn after turn at 'Begin SUPERA' (entrance to A1BF main dipole), is now stationary.

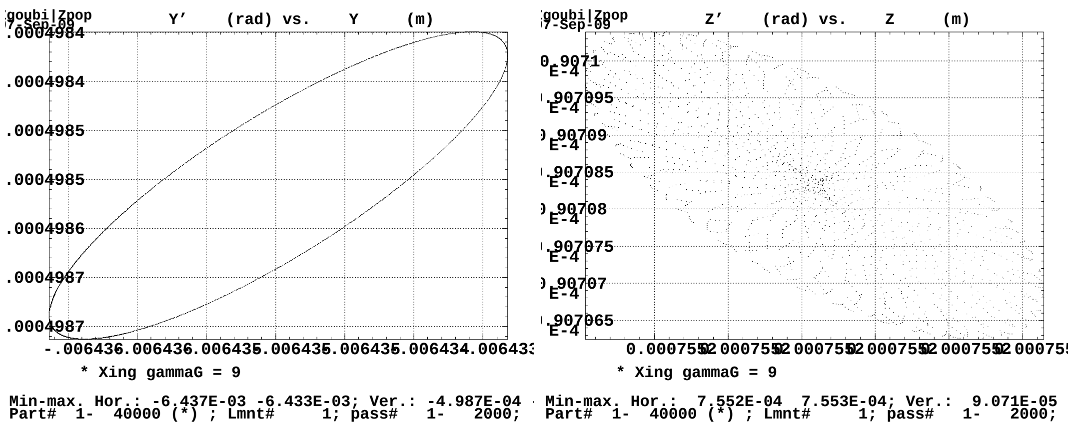



Figure 160: The particle sits on H and V closed orbits - observed at 'Begin SUPERA' (entrance to A1BF main dipole). Namely, the particle describes quasi zero $\epsilon_x$ and $\epsilon_y$ invariants, as shown here.

$$\boxed{\vec{S}_{initial} = \vec{S}_z}$$

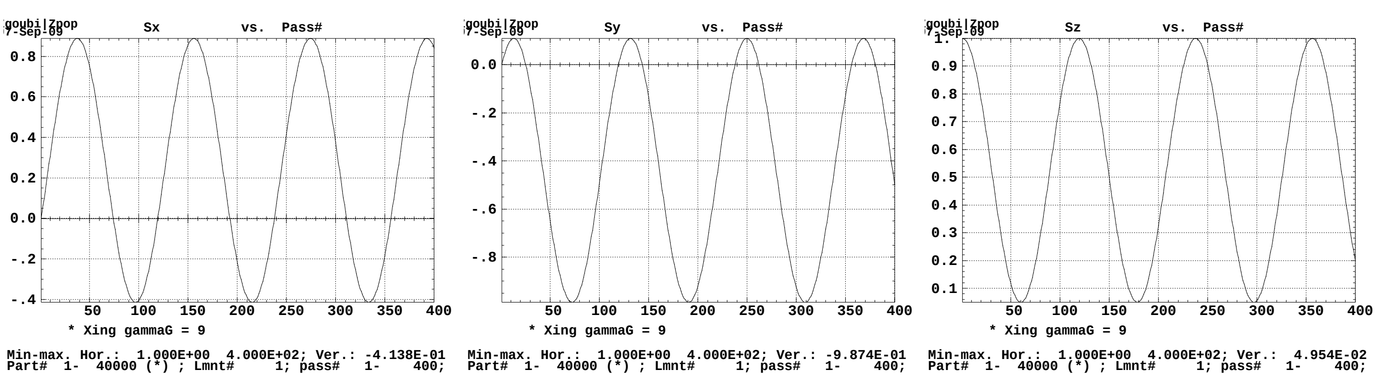



Figure 161: From top to bottom, $S_x$ $S_y$ $S_z$ versus turn number observed at 'Begin SUPERA'. Static case ($T = 3775.00$ GeV), given $\vec{S}_{initial} = \vec{S}_z$.

$\vec{n}_0$ **vector over a turn,** $\vec{S}_{initial} = \vec{S}_{final} \equiv closed\text{-}orbit\ \vec{S}$

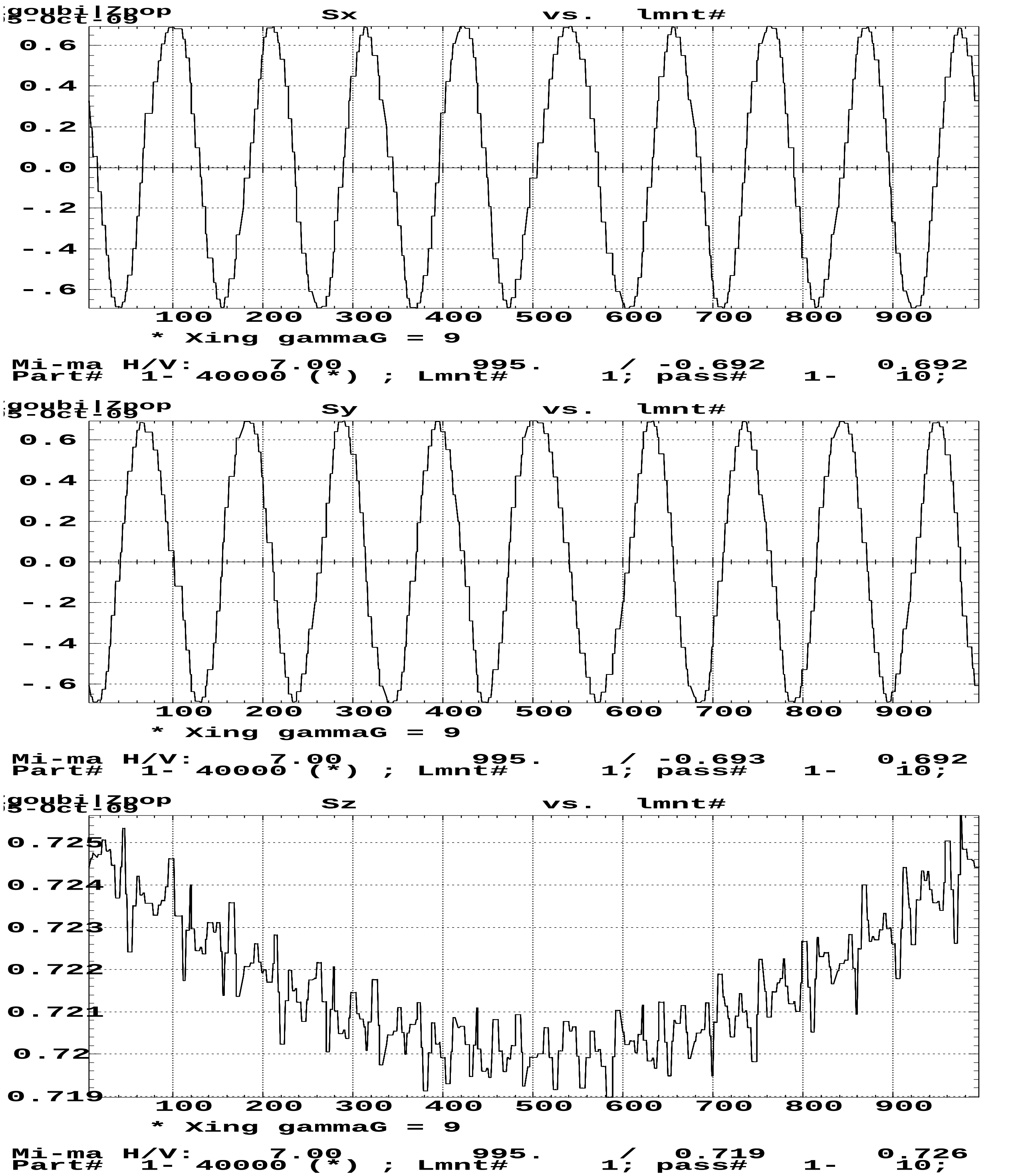



Figure 162: From top to bottom, $S_x$ $S_y$ $S_z$ along machine circumference versus pick-up number (about 990 P-Us over the 807 m circumference). Static case ($T = 3775.00$ GeV), given $\vec{S}_{initial} \equiv closed\text{-}orbit\ \vec{S}$.

# APPENDIX

# A MAD files

Excerpts of MAD files are reproduced, for reference.

## A.1 Command file

```
option,-echo
gam=4.5/1.7928474
mass = 0.93827231
MOM :=  sqrt(gam*gam-1)*mass

call file = "f.constants"
call file = "f.conversions"

call "nosnake.sub"
title "G*gam=4.5  A20snk 2.5T, E20snk 1.5T, A19cor = B1cor"

call file = "f.snkinsert.lat"
call file = "f.lat"
XXX:=0

!TITLE, "AGS SEB MAD,  initiated 2/6/98, KAB"
USE, AGS
beam, particle=proton, gamma=gam, sige=0.001, sigt=1

use, ags
print,#E
twiss, tape, deltap=-0.01,0.0,0.01 save

stop
```

## A.2 “print” file

```
1G*gam=4.5  A20snk 2.5T, E20snk 1.5T, A19cor = B1cor                              "MAD" Version: 8.23/08   Run: 24/08/09  06.33.46
 Linear lattice functions.    TWISS              line: AGS                       range: #S/#E
 Delta(p)/p:    -0.010000     symm: F            super:   1                                                           page     1
 ------------------------------------------------------------------------------------------------------------------------------
      ELEMENT SEQUENCE       I                H O R I Z O N T A L              I                  V E R T I C A L
 pos.  element occ.     dist I    betax  alfax   mux    x(co)  px(co) Dx     Dpx  I   betay  alfay   muy    y(co)  py(co) Dy     Dpy
 no.   name    no.      [m]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]
 ------------------------------------------------------------------------------------------------------------------------------
 begin AGS        1     0.000    19.403 -1.574   0.000-19.5858 -1.363  1.974 0.131   11.696  1.050   0.000  0.0000  0.000  0.000 0.000
 end   AGS        1   807.076    19.403 -1.574   8.918-19.5858 -1.363  1.974 0.131   11.696  1.050   8.750  0.0000  0.000  0.000 0.000
 ------------------------------------------------------------------------------------------------------------------------------
 total length =        807.075641          Qx           =          8.917998          Qy           =          8.750382
 delta(s)     =       -120.036170 mm       Qx'          =        -22.144970          Qy'          =          1.106873
 alfa         =          0.131374E-01      betax(max)   =         22.522370          betay(max)   =         22.897941
 gamma(tr)    =          8.724595          Dx(max)      =          2.416062          Dy(max)      =          0.000000
                                           Dx(r.m.s.)   =          1.886261          Dy(r.m.s.)   =          0.000000
                                           xco(max)     =         21.526346          yco(max)     =          0.000000
                                           xco(r.m.s.)  =         17.795141          yco(r.m.s.)  =          0.000000
 ------------------------------------------------------------------------------------------------------------------------------
1G*gam=4.5  A20snk 2.5T, E20snk 1.5T, A19cor = B1cor                              "MAD" Version: 8.23/08   Run: 24/08/09  06.33.46
 Linear lattice functions.    TWISS              line: AGS                       range: #S/#E
 Delta(p)/p:     0.000000     symm: F            super:   1                                                           page     1
 ------------------------------------------------------------------------------------------------------------------------------
      ELEMENT SEQUENCE       I                H O R I Z O N T A L              I                  V E R T I C A L
 pos.  element occ.     dist I    betax  alfax   mux    x(co)  px(co) Dx     Dpx  I   betay  alfay   muy    y(co)  py(co) Dy     Dpy
 no.   name    no.      [m]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]
 ------------------------------------------------------------------------------------------------------------------------------
 begin AGS        1     0.000    19.785 -1.585   0.000  0.0000  0.000  2.211 0.154   11.701  1.037   0.000  0.0000  0.000  0.000 0.000
 end   AGS        1   807.076    19.785 -1.585   8.711  0.0000  0.000  2.211 0.154   11.701  1.037   8.764  0.0000  0.000  0.000 0.000
 ------------------------------------------------------------------------------------------------------------------------------
 total length =        807.075641          Qx           =          8.710595          Qy           =          8.764380
 delta(s)     =          0.000000 mm       Qx'          =        -22.734051          Qy'          =          1.734352
 alfa         =          0.140063E-01      betax(max)   =         22.917409          betay(max)   =         22.743834
 gamma(tr)    =          8.449647          Dx(max)      =          2.356352          Dy(max)      =          0.000000
                                           Dx(r.m.s.)   =          1.984102          Dy(r.m.s.)   =          0.000000
                                           xco(max)     =          0.000000          yco(max)     =          0.000000
                                           xco(r.m.s.)  =          0.000000          yco(r.m.s.)  =          0.000000
 ------------------------------------------------------------------------------------------------------------------------------
1G*gam=4.5  A20snk 2.5T, E20snk 1.5T, A19cor = B1cor                              "MAD" Version: 8.23/08   Run: 24/08/09  06.33.46
 Linear lattice functions.    TWISS              line: AGS                       range: #S/#E
 Delta(p)/p:     0.010000     symm: F            super:   1                                                           page     1
 ------------------------------------------------------------------------------------------------------------------------------
      ELEMENT SEQUENCE       I                H O R I Z O N T A L              I                  V E R T I C A L
 pos.  element occ.     dist I    betax  alfax   mux    x(co)  px(co) Dx     Dpx  I   betay  alfay   muy    y(co)  py(co) Dy     Dpy
 no.   name    no.      [m]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]  I   [m]    [1]     [2pi]  [mm]   [.001] [m]    [1]
 ------------------------------------------------------------------------------------------------------------------------------
 begin AGS        1     0.000    18.727 -1.324   0.000 20.7717  1.436  2.337 0.162   11.695  1.023   0.000  0.0000  0.000  0.000 0.000
 end   AGS        1   807.076    18.727 -1.324   8.499 20.7717  1.436  2.337 0.162   11.695  1.023   8.784  0.0000  0.000  0.000 0.000
 ------------------------------------------------------------------------------------------------------------------------------
 total length =        807.075641          Qx           =          8.498805          Qy           =          8.784146
 delta(s)     =        126.662810 mm       Qx'          =        -21.581260          Qy'          =          2.394821
 alfa         =          0.149658E-01      betax(max)   =         27.834540          betay(max)   =         22.586341
 gamma(tr)    =          8.174277          Dx(max)      =          2.472667          Dy(max)      =          0.000000
                                           Dx(r.m.s.)   =          2.097844          Dy(r.m.s.)   =          0.000000
                                           xco(max)     =         22.031544          yco(max)     =          0.000000
                                           xco(r.m.s.)  =         18.634115          yco(r.m.s.)  =          0.000000
 ------------------------------------------------------------------------------------------------------------------------------
```

# B Zgoubi data file specimen

## B.1 1-turn first order mapping calculation

Typical zgoubi.dat file including appropriate commands for MATRIX calculation.

```
Data generated by searchCO
'OBJET'                                                             1
        1000.0
5
 .001 .001 .001 .001 0. .0001
 -6.431975E-01 -4.982951E-01  0.0E+00  0.0  0.0 1.000E+00 'o'
 'PARTICUL'                                                         2
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
 'SCALING'                                                          3
 0  3
  MULTIPOL SBEN
  2
  7.50690875556628   7.50690875556628
  1                  99999
  MULTIPOL QUAD
  2
  7.50690875556628   7.50690875556628
  1                  99999
   'PICKUPS'                                                        4
   1
   #S
   'FAISTORE'                                                       5
   b_zgoubi.fai   #S
   1
   'MARKER'  #S                                                     6
 'MARKER'   MARK      BSUPERA                                       7
 'MULTIPOL' SBEN      A1BF                                          8
 0  .Dip
 200.6554 10. 0.11712499 0.04848519 -0.00050563 0 0 0 0 0 0 0
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|20|20    Dip  A1BF
           3   0.0000        0.0000      -1.1751150450E-002
 'DRIFT'    DRIF      D2S                                           9
      60.9515
 'MULTIPOL' SBEN      A2BF                                         10
 0  .Dip
  200.6554  10. 0.11712499 0.04848519  -0.00050563 0 0 0 0 0 0 0
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|20|20    Dip  A2BF
           3   0.0000        0.0000      -1.1751150450E-002
 'DRIFT'    DRIF      DPUE                                         11
      28.7000
 'MARKER'   MARK      PUE_A02                                      12
 'MULTIPOL' HKIC      DHCA02                                       13
 0  .kicker
   0.1000E-03 10.  0.0E+00    0 0 0 0 0 0 0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.0000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'MULTIPOL' VKIC      DVCA02                                       14
 0  .kicker
   0.1000E-03 10.  0.0E+00    0 0 0 0 0 0 0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      D2TX                                         15
      32.2485
 'MULTIPOL' SBEN      A3CD                                         16
 0  .Dip
  238.7522  10. 0.11712611 -0.04865238 -0.00043783 0 0 0 0 0 0 0
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|24|20    Dip  A3CD
           3   0.0000        0.0000      -1.3982515350E-002
 'DRIFT'    DRIF      DSQ                                          17
      37.1047
 'MULTIPOL' QUAD      QHFV                                         18
 0  .Quad
   39.0880 10.    0.00000    0.00000 0 0 0 0 0 0 0 0 0.0
 0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
 0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #30|39|30    Quad  QHFV
 1 0. 0. 0.
 'MULTIPOL' QUAD      QPOL                                         19
 0  .Quad
    39.0880 10.    0.00000    0.00000 0 0 0 0 0 0 0 0 0.0
 0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
 0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #30|39|30    Quad  QPOL
 1 0. 0. 0.
 'DRIFT'    DRIF      DSQ                                          20
      37.1047
...................
...................
...................
 'DRIFT'    DRIF      DPUE                                        982
      28.7000
 'MARKER'   MARK      PUE_L18                                     983
 'MULTIPOL' HKIC      DHCL18                                      984
 0  .kicker
   0.1000E-03 10.  0.0E+00    0 0 0 0 0 0 0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.0000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'MULTIPOL' VKIC      DVCL18                                      985
 0  .kicker
   0.1000E-03 10.  0.0E+00    0 0 0 0 0 0 0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  1.570796327 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      D2TX                                        986
      32.2485
 'MULTIPOL' SBEN      L19BD                                       987
 0  .Dip
 200.6554 10. 0.11712499 -0.04843927 -0.00050017 0 0 0 0 0 0 0
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|20|20    Dip  L19BD
           3   0.0000        0.0000      -1.1751150450E-002
 'DRIFT'    DRIF      D2S                                         988
      60.9515
 'MULTIPOL' SBEN      L20BD                                       989
 0  .Dip
  200.6554 10. 0.11712499 -0.04843927 -0.00050017 0. 0. 0. 0. 0. 0 0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.0000 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|20|20    Dip  L20BD
           3   0.0000        0.0000      -1.1751150450E-002
 'DRIFT'    DRIF      DSG10                                       990
      31.7339
 'MULTIPOL' HKIC      SML20                                       991
 0  .kicker
   0.2413E+03 10.  0.0E+00    0 0 0 0 0 0 0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.0000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      DSG10                                       992
      31.7339
 'MARKER'   MARK      ESUPERL                                     993
 'MATRIX'                                                         994
  1  11
 'FAISCEAU'                                                       995
 'END'                                                            996
```

## B.2 Zgoubi files, $\vec{n}_0$ vector search using FIT

### zgoubi.dat including FIT input :

```
Xing gammaG = 9
'OBJET'                                                                                                                   1
15.4071e3   14.95063e3
2
1  1
 -0.64333404 -0.49841318  7.55234854E-02  9.07085314E-02   0.0E+00  1.    'p'
1
 'SCALING'                                                                                                                2
 1  3
  MULTIPOL VKIC
  -1
  15.4071                         scales the vertical kick
  1
  MULTIPOL SBEN
  -1
  15.4071
  1
  MULTIPOL QUAD
  -1
  15.4071
  1
 'PARTICUL'                                                                                                               3
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
'FAISCEAU'                                                                                                                4
 'SPNTRK'                                                                                                                 5
  4.1
  0.3222862708 -0.6092011735 0.7241015089
   'MARKER'  #Start                                                                                                       6
 'MARKER'   MARK      BSUPERA                                                                                             7
 'MULTIPOL' SBEN      A1BF                                                                                                8
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|200|20    Dip  A1BF
            3   0.00000000000000        0.00000000000000      -1.175115045000000E-002
 'DRIFT'    DRIF      D2S                                                                                                 9
      60.9515
.........................
.........................
.........................
 'DRIFT'    DRIF      DSG10                                                                                             990
      31.7339
 'MULTIPOL' HKIC      SML20                                                                                             991
 0  .kicker
   0.2413E+03 10.00  0.000000E+00    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      DSG10                                                                                             992
      31.7339
 'MARKER'   MARK      ESUPERL                                                                                           993
   'MARKER'  #End                                                                                                       994
'SPNPRT'                                                                                                                995
'FIT'                                                                                                                   996
  3            Variables :
 5 10  0   2   Sx_0
 5 11  0   2   Sy_0
 5 12  0   1   Sz_0
  3            Constraints :
 10.1 1 1  994  0. .3 0   Sx  at end
 10.1 1 2  994  0. .3 0   Sy  at end
 10.1 1 3  994  0. 1. 0   Sz  at end
 'END'                                                                                                                  997
```

### zgoubi.res including FIT output :

```
    996  FIT
           variable #           1       IR =           5 ,   ok.
           variable #           1       IP =          10 ,   ok.
           variable #           2       IR =           5 ,   ok.
           variable #           2       IP =          11 ,   ok.
           variable #           3       IR =           5 ,   ok.
           variable #           3       IP =          12 ,   ok.
           constraint #           1       IR =         994 ,   ok.
           constraint #           2       IR =         994 ,   ok.
           constraint #           3       IR =         994 ,   ok.

                     FIT  variables  in  good  order,  FIT  will proceed.

 STATUS OF VARIABLES  (Iteration #     0)
 LMNT  VAR  PARAM  MINIMUM     INITIAL         FINAL         MAXIMUM      STEP
    5    1     10  -0.322       0.328      0.3278000720      0.967      9.764E-15
    5    2     11   -1.83      -0.606     -0.6064336572      0.609      1.729E-14
    5    3     12    0.00       0.724      0.7244206873       1.45      1.081E-14
 STATUS OF CONSTRAINTS
 TYPE  I   J  LMNT#      DESIRED         WEIGHT          REACHED         KI2         *  Parameter(s)
```

```
  10   1   1   994       0.0000         0.3000        1.2911894E-13   2.0915E-03     *  0 :
  10   1   2   994       0.0000         0.3000        1.1151080E-12   0.1560         *  0 :
  10   1   3   994       0.0000          1.000        8.6353147E-12   0.8419         *  0 :
```

## B.3 Zgoubi files, ring closed orbit search using FIT

### zgoubi.dat including FIT input :

```
Data generated by searchCO
'OBJET'                                                                                                                1
   1000.
5
 .001 .001 .001 .001 0. .001
 -6.431975E-01 -4.982951E-01  0.0E+00  0.0E+00  0.0E+00  1.00000000E+00 'o'
 'MARKER'   MARK      BSUPERA                                                                                          2
 'MULTIPOL' SBEN      A1BF                                                                                             3
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|20|20    Dip  A1BF
          3   0.00000000000000        0.00000000000000      -1.175115045000000E-002
 'DRIFT'    DRIF      D2S                                                                                              4
      60.9515
..................
..................
..................
 'MULTIPOL' HKIC      SML20                                                                                          975
 0  .kicker
   0.2413E+03 10.00  0.000000E+00    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      DSG10                                                                                          976
      31.7339
 'MARKER'   MARK      ESUPERL                                                                                        977
 'MATRIX'                                                                                                            978
  1  11
 'FAISCEAU'                                                                                                          979
 'FIT'                                                                                                               980
 3
 1 30 0 3
 1 31 0 3
 1 35 0 .1
 3
 3.1  1 2 978 0.      1 0      x_co
 3.1  1 3 978 0.      1 0      xp_co
 3    1 6 978 80707.6 1 0  co length
 'END'                                                                                                               981
```

### zgoubi.res including FIT output :

```
 STATUS OF VARIABLES  (Iteration #     0)
 LMNT  VAR  PARAM  MINIMUM     INITIAL         FINAL                        MAXIMUM      STEP
    1    1     30  -0.256      -7.034E-02  -7.0344865763E-02 = x_co (cm)    0.128      3.098E-13
    1    2     31  -0.359      -9.439E-02  -9.4388984947E-02 = x'_co (cm)   0.180      3.926E-13
    1    3     35   0.903        1.00        1.002797547     = p/p_0        1.10      2.127E-13
 STATUS OF CONSTRAINTS
 TYPE  I   J  LMNT#       DESIRED           WEIGHT         REACHED         KI2         *  Parameter(s)
   3   1   2   978      0.0000000E+00     1.0000E+00    1.3279162E-09   6.9766E-01     *  0 :
   3   1   3   978      0.0000000E+00     1.0000E+00    8.0138363E-10   2.5409E-01     *  0 :
   3   1   6   978      8.0707564E+04     1.0000E+00    8.0707564E+04   4.8257E-02     *  0 :


Path length of particle  1 :   80707.564     cm

                  TRANSFER  MATRIX  ORDRE  1  (MKSA units)
          0.732535        -16.3490         0.00000         0.00000         0.00000         2.92379
          0.145067        -1.87262         0.00000         0.00000         0.00000        0.118147
           0.00000         0.00000       -0.909176        -11.6183         0.00000         0.00000
           0.00000         0.00000        0.175032         1.13689         0.00000         0.00000
         -0.338693         3.56095         0.00000         0.00000         1.00000         11.6934
           0.00000         0.00000         0.00000         0.00000         0.00000         1.00000

      First order sympletic conditions (expected values = 0) :
        -4.5006E-05   -5.9300E-05     0.000         0.000         0.000         0.000

       Beam  matrix  (beta/-alpha/-alpha/gamma) and  periodic  dispersion  (MKSA units)
          19.898601     1.585380     0.000000     0.000000     0.000000     2.059607
           1.585380     0.176567     0.000000     0.000000     0.000000     0.145139
           0.000000     0.000000    11.694319    -1.029730     0.000000     0.000000
           0.000000     0.000000    -1.029730     0.176183     0.000000     0.000000
           0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
           0.000000     0.000000     0.000000     0.000000     0.000000     0.000000

                                   Betatron  tunes
                    NU_Y = 0.65346381         NU_Z = 0.76816038
```