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Output from "CYCLOTRON" data treatment

1 Analytical model

Cyclotron provides a model of a dipole field. The field along the particle's trajectory is computed as the particle motion proceeds, by using the magnet's geometrical boundaries: At any position P(R,θ) along the particle trajectory (see Fig.1), the value of the vertical component of the mid-plane field is calculated using:

$$B_Z(R,\theta) = B_{norm} \times \mathscr{F}(R,\theta) \times \mathscr{R}(R) \tag{1}$$

(according to the Enge fringe model) where:

- $\mathscr{R}(R) = B_0 + B_1 \times R + B_2 \times R^2 + B_3 \times R^3 + B_4 \times R^4$,
- B_{norm} is a normalization coefficient,
- $\mathscr{F}(R,\theta)$ is the fringe field coefficient, given by:

$$\mathscr{F}(R,\theta) = \mathscr{F}_{entr}(R,\theta) \times \mathscr{F}_{exit}(R,\theta) = \frac{1}{1+exp(P_{entr}(d_{entr}))} \times \frac{1}{1+exp(P_{exit}(d_{exit}))} \tag{2}$$

where

$$P(d) = C_0 + C_1\left(\frac{d}{g}\right) + C_2\left(\frac{d}{g}\right)^2 + C_3\left(\frac{d}{g}\right)^3 + C_4\left(\frac{d}{g}\right)^4 + C_5\left(\frac{d}{g}\right)^5$$

and d is the distance from the Effective Field Boundary (EFB) either at the entrance or at the exit of the magnet (d_{entr} and d_{exit} as shown in Fig.1)

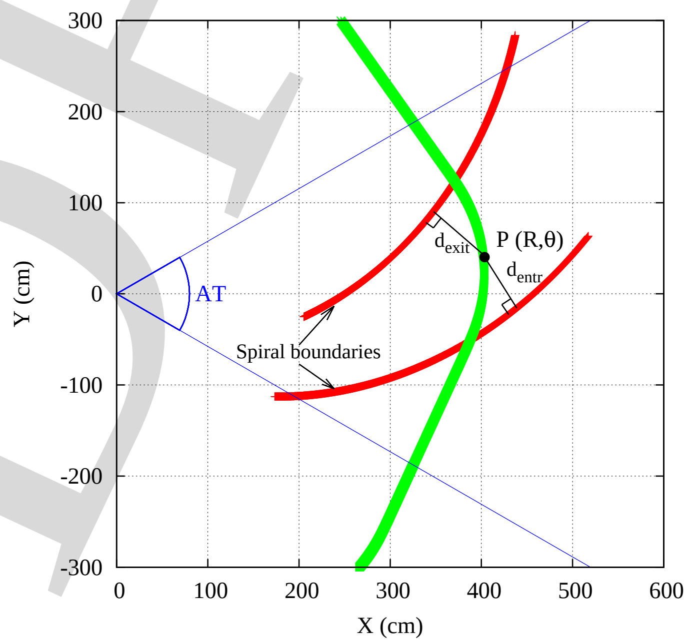


Figure 1: "CYCLOTRON" definition of the EFB: the trajectory of the moving particle (anti-clockwise motion) is in green while the EFB are in red. "AT" is the total angle of the sector ($\frac{2\pi}{AT}$ defines the number of sectors of the entire ring.)

The effective field boundaries are modelled by a logarithmic spiral for which the angle ξ is allowed to increase radially, namely:

$$r = r_0 \times \exp\left(\frac{\theta+\omega}{\tan(\xi(r))}\right) \tag{3}$$

where $\xi(r) = \xi_0 + \xi_1 \times r + \xi_2 \times r^2 + \xi_3 \times r^3$, θ is the azimuthal angle (origin θ=0) and ω is a parameter used to position the EFB with respect to the azimuthal position θ=0.

According to this model, the magnet gap is also allowed to vary: g is given as a function of the radius by:

$$g(r) = g_0 + g_1 \times r + g_2 \times r^2 \tag{4}$$

The field is then extrapolated off median plane by means of Taylor series: for that the median plane anti-symmetry is assumed and the Maxwell equations are accommodated.

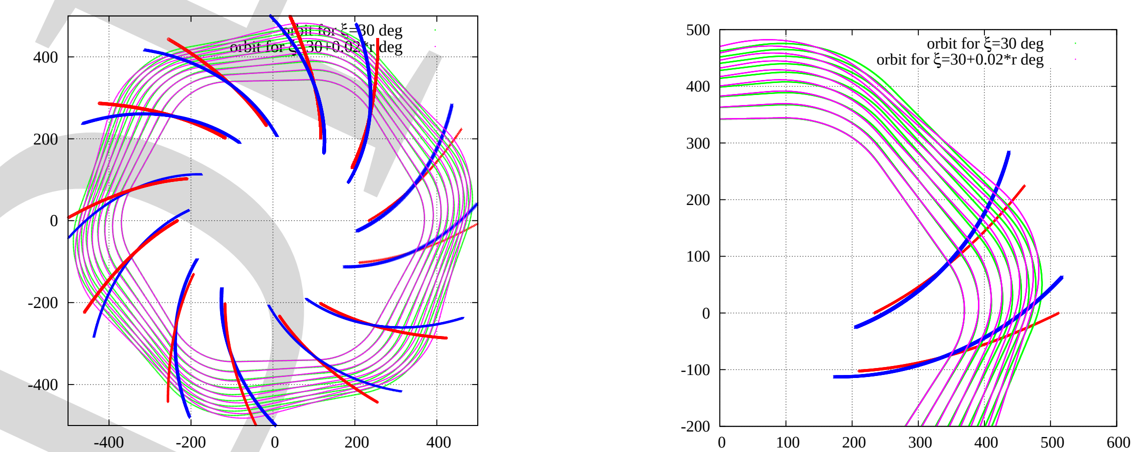
NB: - The EFB can also be straight, in which case it is defined by the cartesian equation: ax+by+c=0.
- "CYCLOTRON" allows the overlapping of 5 such dipole fields. This follows the method described in [1]

2 Example of a 6-sector cyclotron model:

In the case of a cyclotron machine, the isochronicity is a crucial point: Because the revolution time has to be constant ($f_{rev} = \frac{qB}{2\pi\gamma m_0}$), this implies that the radial dependence of the field must be proportional to γ, so that $\mathscr{R}(\overline{R}) \propto \gamma(\overline{R})$, where $\overline{R}$ is the average radius of the orbit. Since $f_{rev} = \frac{v}{C}$, where C is the path length of the particle for one closed orbit, we obtain, with a good approximation, that $R \propto \beta$. Thus,

$$\mathscr{R}(R) \approx \frac{1}{\sqrt{1-(\frac{R}{R_0})^2}} \tag{5}$$

We searched for the closed orbits ensuring the 6-fold symmetry of the machine. The results of the orbits as well as the magnetic field are shown in Fig. 2 and 3 respectively.



(a) Plot of the orbits as well as the EFB for the entire ring.

(b) Zoom in one sector (see Fig2(a))

Figure 2: Plot of the closed orbits of a 6-sector cyclotron machine for two different configurations, whether or not the spiral angle ξ is allowed to vary: in red the EFB has a constant spiral angle ξ=30 deg (corresponding orbit shown in green) while in, blue, the EFB spiral angle is allowed to vary linearly with the radius: $\xi = 30 + 0.02 \times r$.

[1] F. Lemuet, F. Meot, Developements in the ray-tracing code Zgoubi for 6-D multiturn tracking in FFAG rings, NIM A 547 (2005) 638-651.

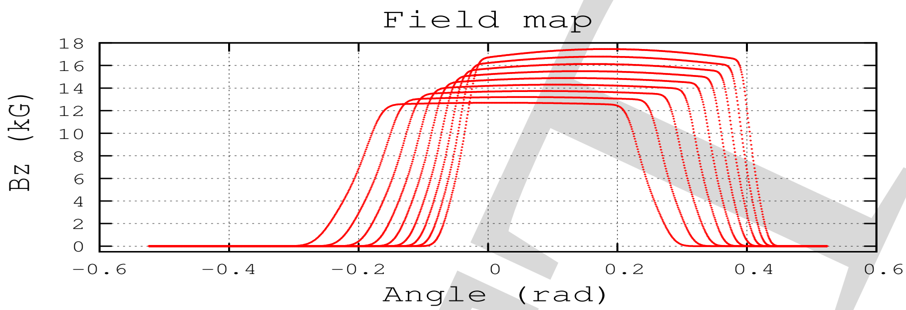


Figure 3: Bz (i.e. vertical) component of the magnetic field along trajectories, step by step, across "CYCLOTRON" (case ξ = 30 deg).

3 Application to the case of the PSI main ring cyclotron

In order to obtain the equation of the EFB for the PSI ring cyclotron, we proceed as follows:
1) Tracking using the field map in polar coordinates is performed, where the closed orbits and the magnetic field along the trajectories are computed.
2) The magnetic effective field length is computed at the entrance and the exit of the magnet (EQ 6) for each energy (which corresponds to a different closed orbit). For example, for the closed orbit corresponding to an energy of 175MeV (arc $\widehat{ABC}$ as shown in Fig.4), one has [2]

$$Leff_{ent} = \frac{\int_A^B B_z dl}{B_z^{max}}, \qquad Leff_{ex} = \frac{\int_B^C B_z dl}{B_z^{max}} \tag{6}$$

The equation of the EFB is then obtained by fitting the data points at the entrance and the exit of the magnet to the equation of a spiral which gives, in EQ3,

- At entrance: r0 = 276 cm , $\xi(deg) = 2 + 12.10^{-3} \times r + 75.10^{-6} \times r^2$, $\omega = -8.5(deg)$
- At exit: r0 = 276 cm , $\xi(deg) = 3.5 + 35.10^{-3} \times r + 3.10^{-8} \times r^3$, $\omega = 11(deg)$

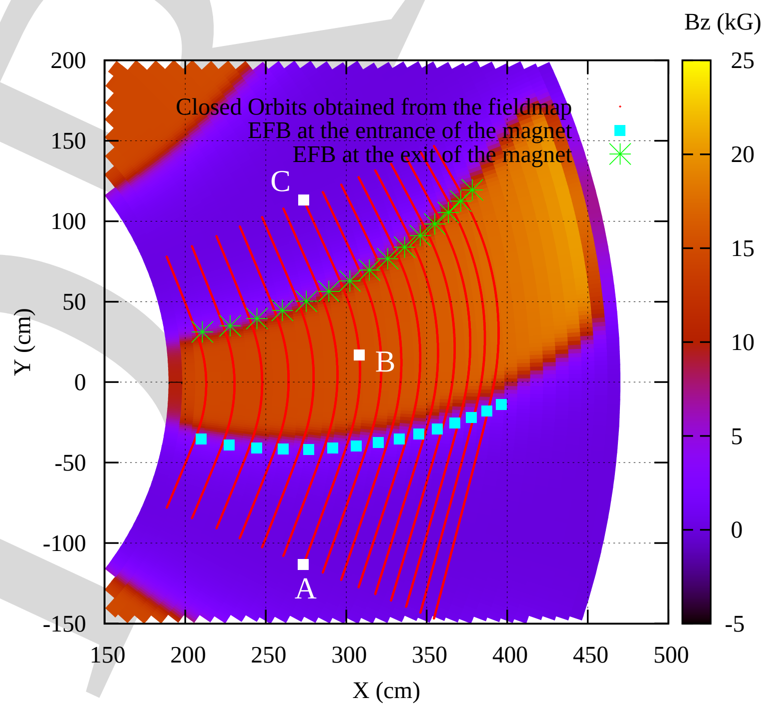


Figure 4: EFB as obtained from the field calculation along the different closed orbits. Here B corresponds to B_z^{max} in EQ. 6.

[2]T.Planche et al, Design of a prototype gap shaping spiral dipole for a variable energy protontherapy FFAG, NIM A 604 (2009) 435-442

3) The radial field law $\mathscr{R}(R)$ is then obtained by fitting B_z^{max} for the different closed orbits as a function of the radius, which gives the coefficient of the polynomial $(B_0, B_1, B_2, B_3, B_4)$.
4) Finally, the polynomial coefficients $(C_0, .., C_5)$ used to determine the fringe field coefficient $\mathscr{F}(R,\theta)$ are obtained using a fitting method: we choose a single closed orbit to fit (in the resent we choose the 137 MeV energy) (see Fig.5). We fill "CYCLOTRON" in the "zgoubi.dat" file with the definition of the EFBs and provide the closed orbit that was obtained from the previous calculation, using the field map. We run zgoubi and print the results in a file containing all the parameters required to compute B_z from EQ.1 (namely d_{entr}, d_{exit} and R). The data to fit consists of B_z obtained from the measured median plane field map. A python script is used to search for the optimum fit parameters, based on updated calls to zgoubi execution, with updated fringe field coefficients.

Once all these parameters determined, tracking can be performed using "CYCLOTRON" and some of the results are shown below. In Fig.5, the EFBs and the 137 MeV trajectory are superposed to the measured median plane field map of the cyclotron (in order to check the correct behaviour of the semi-analytical "CYCLOTRON" model).
A comparison of the orbit trajectories as well as the magnetic field with both calculations is finally shown in Fig.6 and 7 respectively: using the fringe field parameters obtained from the 137 MeV orbit fit, the differences obtained are less than 0.03 % while for orbits with higher energies, the differences can go up to 0.2 %. These results were expected given that the fringe field coefficients were obtained from a fitting method which only minimizes the differences between the orbits at 137 MeV. To improve this method, the Fringe field coefficients should be allowed to vary as a function of the radius. However, the aim of the model is not to reproduce the fieldmap calculations, but to be able to tweak few lattice parameters in order to come up, later on with an optimized design that can be cross-checked with OPERA calculations.
The python fit procedure stops when the "CYCLOTRON" $B_{cyc}(\theta)$ has to converged to the field map $B_{meas}(\theta)$

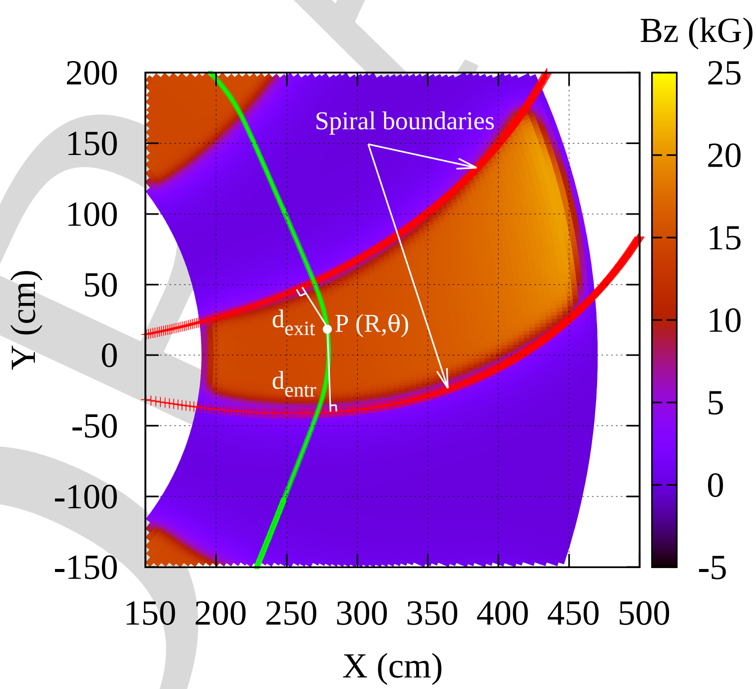


Figure 5: Orbit trajectory as obtained from "CYCLOTRON" model with some geometrical parameters used for the tracking.

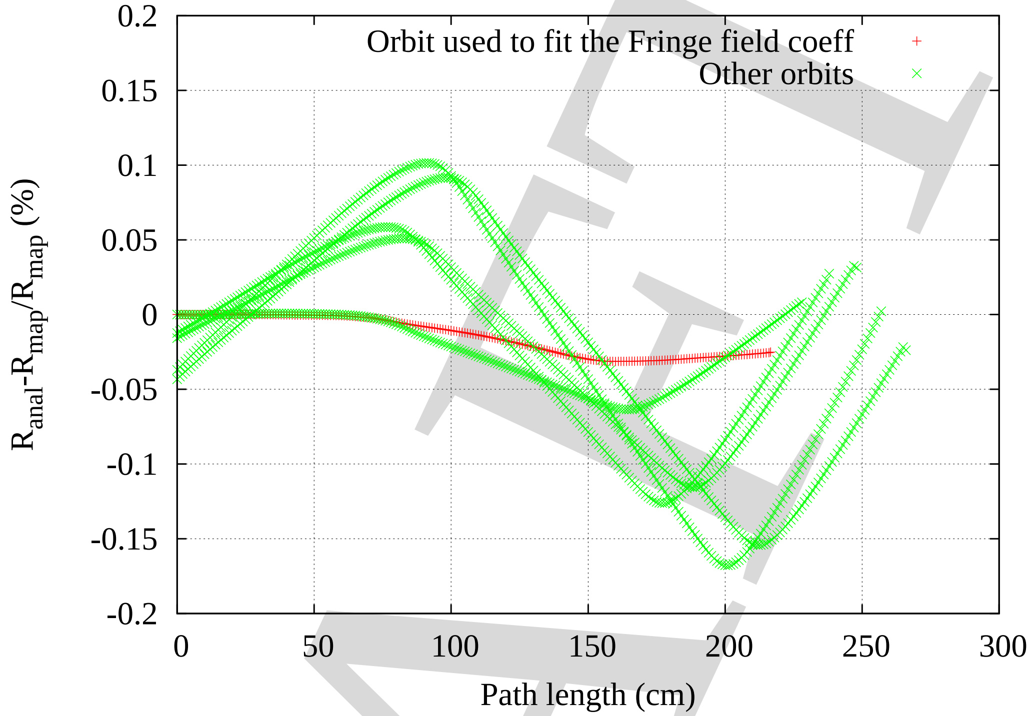


Figure 6: Comparison of the trajectories obtained from the tracking using the analytical model "CYCLOTRON" and the fieldmap using "POLARMES": the relative error is less than 0.03 % for the 137 MeV fitted orbit, while for higher energies, (156, 176, 196, 218 and 240 MeV), the differences can go up to 0.2 %.

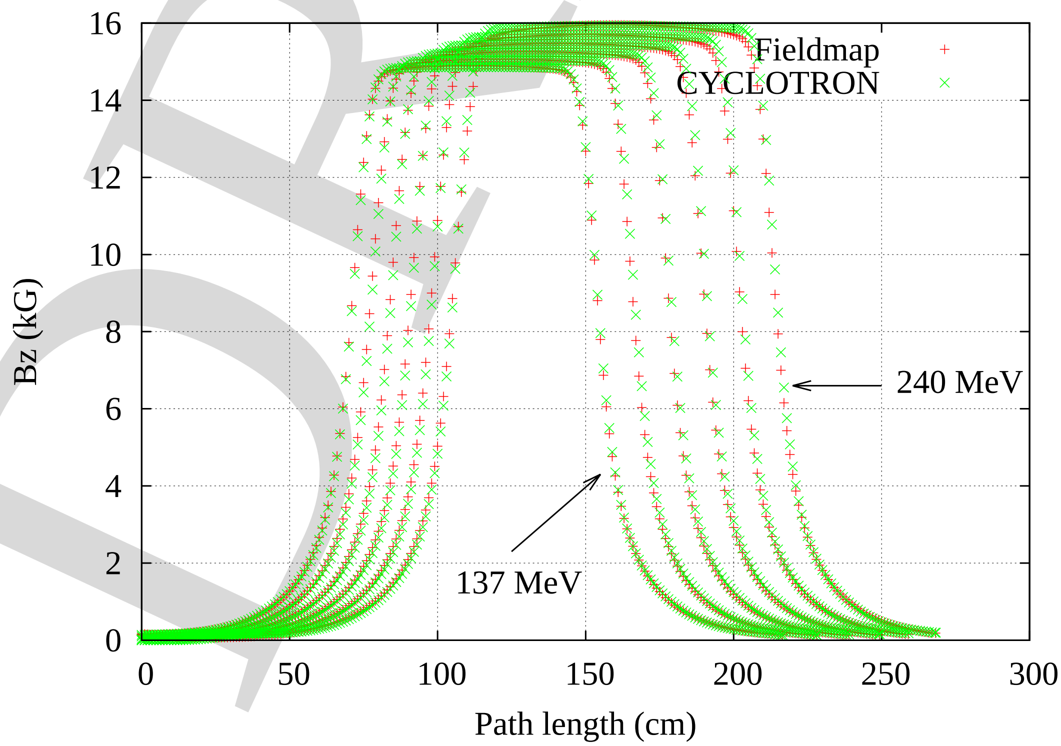


Figure 7: Comparison of the magnetic field along different trajectories (137, 156, 176, 196, 218 and 240 MeV) obtained from the tracking using the analytical model "CYCLOTRON" and the fieldmap ("POLARMES").